BER for QAM

16 QAM

$$bits(x) := floor\left(\frac{\log(x)}{\log(2)} + 0.1\right)$$

$$erfc(x) := 1 - erf(x)$$

$$bits(16) = 4$$

BPSK case added at the end
26 October 2001

$$Q(x) := \frac{1}{2}\cdot\left(1 - erf\left(\frac{x}{\sqrt{2}}\right)\right)$$

$$P_{b16QAM}(EsNo) := \frac{1}{2}\cdot Q\left(\sqrt{\frac{EsNo}{5}}\right) + \frac{1}{2}\cdot Q\left(3\cdot\sqrt{\frac{EsNo}{5}}\right)$$

From Modern Quadrature Amplitude Modulation by W.T. Webb, L. Hanzo, Pentech Press, 1994, page 161

$$SNR_{bit\_dB} := 15$$

EsNo is the AVERAGE symbol to noise ratio

$$P_{b16QAM}\left[10^{0.1\cdot(SNR_{bit\_dB})}\cdot bits(16)\right] = 1.228\times 10^{-7}$$ **Bit Error Rate**

From Digital Communications, Proakis, Wiley, 2nd Ed., 1989, page 282

Symbol Error Rate

$$P_s(\gamma_b, M, k) := 2\cdot\left(1 - \frac{1}{\sqrt{M}}\right)\cdot erfc\left[\sqrt{\frac{3\cdot k\cdot\gamma_b}{2\cdot(M-1)}}\right]\cdot\left[1 - \frac{1}{2}\cdot\left(1 - \frac{1}{\sqrt{M}}\right)\cdot erfc\left[\sqrt{\frac{3\cdot k\cdot\gamma_b}{2\cdot(M-1)}}\right]\right]$$ (equ. 4.2.144)

Here, γ_b is the average SNR per bit
k is the number of bits per symbol
M is the total number of points in the square QAM constellation (256)

$$P_s\left(10^{0.1\cdot SNR_{bit\_dB}}, 256, 8\right) = 0.15215195$$

$$\frac{3}{2}\cdot erfc\left(\sqrt{\frac{2}{5}\cdot 10^{0.1\cdot SNR_{bit\_dB}}}\right) = 7.367\times 10^{-7}$$

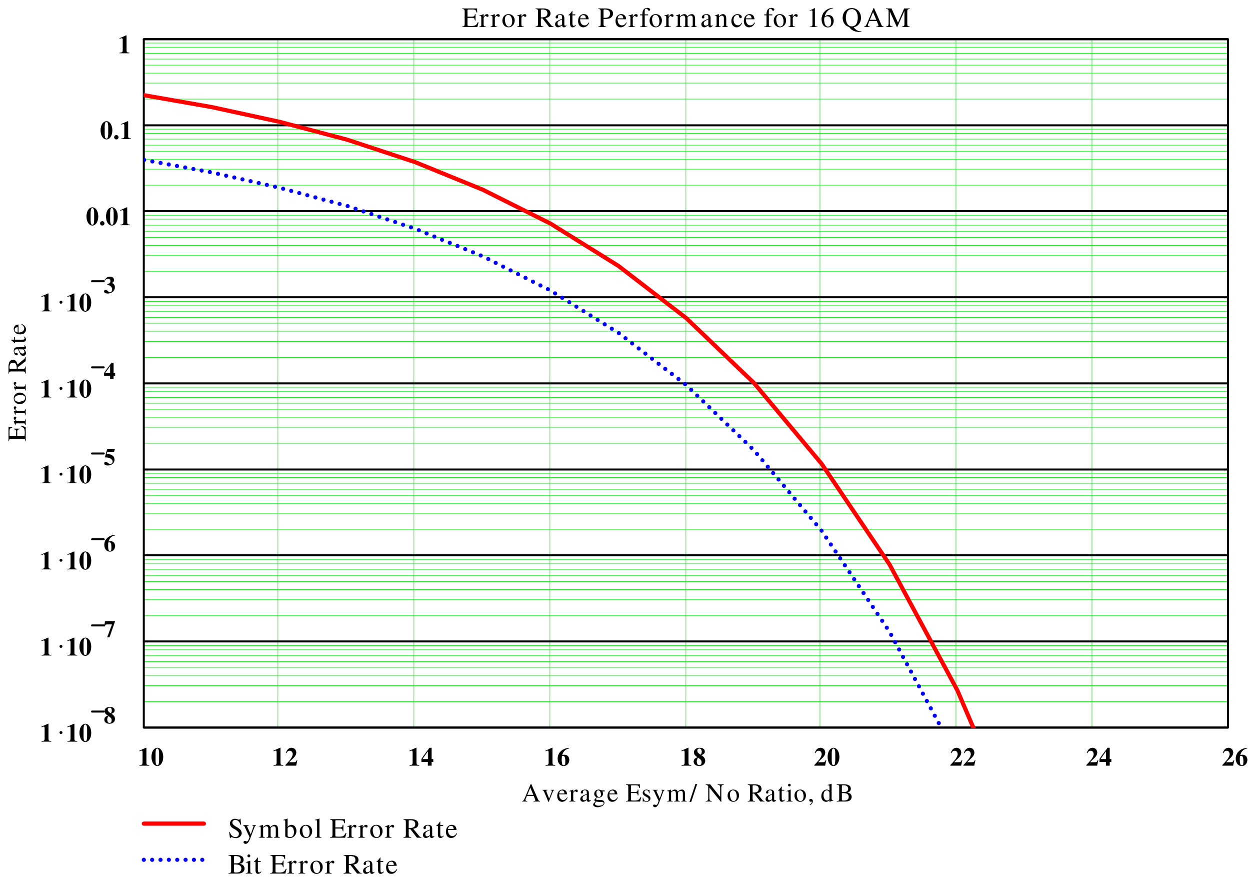
$kb := 0 .. 15$

$EsNo_{dB_{kb}} := 10 + kb$

$EbNo_{dB_{kb}} := EsNo_{dB_{kb}} - 10 \cdot \log(bits(16))$

$ps_{kb} := P_s\left(\frac{10^{0.1 \cdot EsNo_{dB_{kb}}}}{4}, 16, 4\right)$

$pb_{kb} := P_{b16QAM}\left(10^{0.1 \cdot EsNo_{dB_{kb}}}\right)$



Derived Phase Noise Impact for Square QAM Constellations

$$\text{AvEnergyPerSymbol}(\text{CSize}) := \left| \begin{array}{l} \text{Nbits} \leftarrow \text{floor}\left(\frac{\log(\text{CSize})}{\log(2)} + 0.1\right) \\ \text{RailLevels} \leftarrow 2^{0.5\cdot\text{Nbits}} \\ \text{sum} \leftarrow 0 \\ \text{for } ii \in 0 .. \frac{\text{RailLevels}}{2} - 1 \\ \quad \text{for } jj \in 0 .. \frac{\text{RailLevels}}{2} - 1 \\ \quad\quad \text{sum} \leftarrow \text{sum} + \left[\left(\frac{2\cdot ii + 1}{2}\right)^2 + \left(\frac{2\cdot jj + 1}{2}\right)^2\right] \\ \text{sum} \leftarrow \frac{4\cdot\text{sum}}{\text{CSize}} \end{array} \right.$$

$$\text{AvEnergyPerSymbol}(256) = 42.5$$

Assumes square QAM constellation.

Adds up power for all of the constellation points in the upper right-hand quadrant, multiplies this sum by 4 to get the total for the entire constelllation, and then divides by the total number of constellation points.

Rectangular spacing between constellation points assumed to be d= 1.

$$P_{sym}(\theta_n, SNRdB_{bit}, CSize) := \left| \begin{array}{l} Nbits \leftarrow floor\left(\frac{\log(CSize)}{\log(2)} + 0.1\right) \\ RailLevels \leftarrow 2^{\frac{Nbits}{2}} \\ snr_{bit} \leftarrow 10^{0.1 \cdot SNRdB_{bit}} \\ E \leftarrow AvEnergyPerSymbol(CSize) \\ \sigma \leftarrow \sqrt{\frac{E}{2 \cdot snr_{bit} \cdot Nbits}} \\ sum \leftarrow \sum_{k=0}^{\frac{RailLevels}{2}-1} erfc\left[\frac{\frac{1}{2} - \frac{(2 \cdot k + 1)}{2} \cdot \sin(\theta_n)}{\sigma \cdot \sqrt{2}}\right] \\ sum \leftarrow sum + \sum_{k=0}^{\frac{RailLevels}{2}-1} erfc\left[\frac{\frac{1}{2} + \frac{(2 \cdot k + 1)}{2} \cdot \sin(\theta_n)}{\sigma \cdot \sqrt{2}}\right] \\ sum \leftarrow sum \cdot 2 \cdot \left(\frac{RailLevels - 1}{RailLevels^2}\right) \end{array} \right.$$

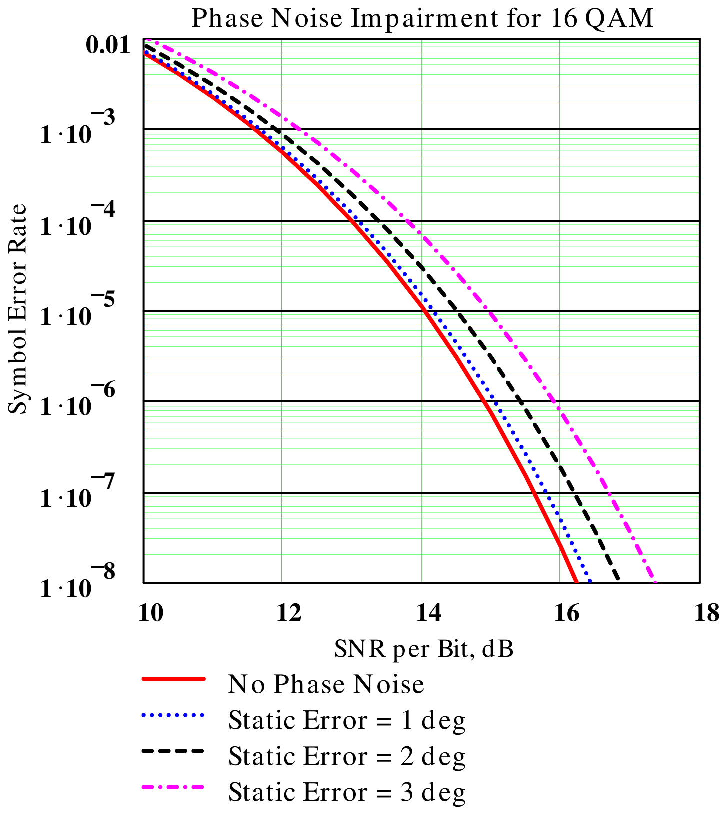
$pp := 0 .. 50$

$SNR_{bitdB_{pp}} := 2 + pp \cdot 0.5$

16 QAM Situation

$y1_{pp} := P_{sym}\left(0, SNR_{bitdB_{pp}}, 16\right)$ $\quad y2_{pp} := P_{sym}\left(\frac{1}{180} \cdot \pi, SNR_{bitdB_{pp}}, 16\right)$

$y3_{pp} := P_{sym}\left(\frac{2}{180} \cdot \pi, SNR_{bitdB_{pp}}, 16\right)$ $\quad y4_{pp} := P_{sym}\left(\frac{3}{180} \cdot \pi, SNR_{bitdB_{pp}}, 16\right)$

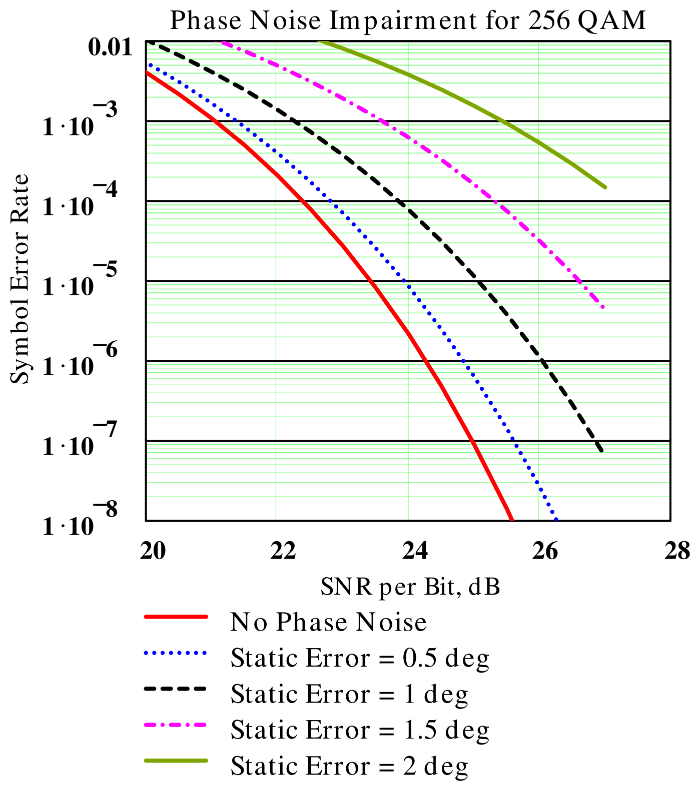


256 QAM Situation

$$y1_{pp} := P_{sym}\left(0, SNR_{bitdB_{pp}}, 256\right)$$

$$y2_{pp} := P_{sym}\left(\frac{0.5}{180}\cdot\pi, SNR_{bitdB_{pp}}, 256\right) \qquad y4_{pp} := P_{sym}\left(\frac{1.5}{180}\cdot\pi, SNR_{bitdB_{pp}}, 256\right)$$

$$y3_{pp} := P_{sym}\left(\frac{1}{180}\cdot\pi, SNR_{bitdB_{pp}}, 256\right) \qquad y5_{pp} := P_{sym}\left(\frac{2.0}{180}\cdot\pi, SNR_{bitdB_{pp}}, 256\right)$$



64 QAM Situation

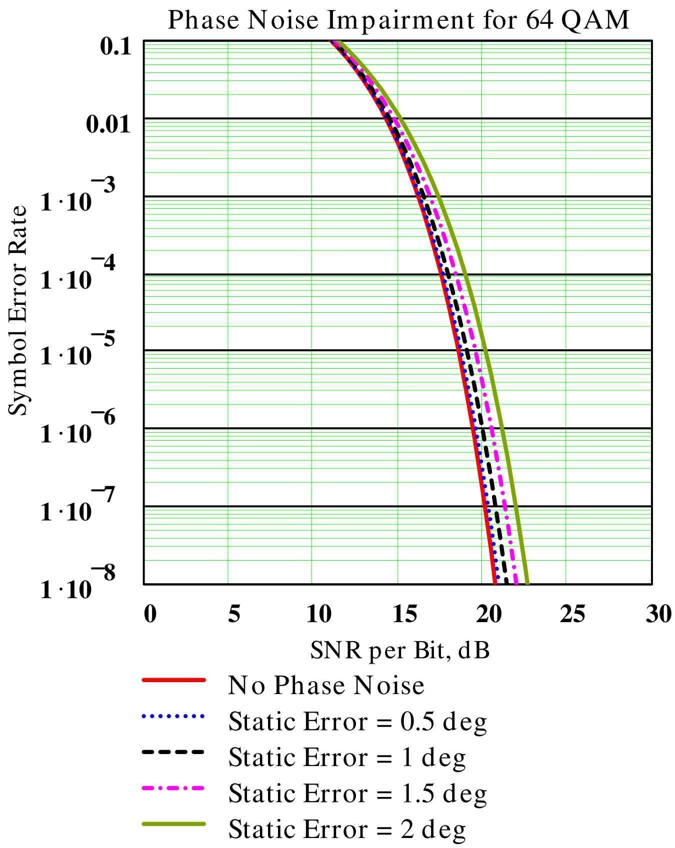
$$y1_{pp} := P_{sym}\left(0, SNR_{bitdB_{pp}}, 64\right)$$

$$y2_{pp} := P_{sym}\left(\frac{0.5}{180}\cdot\pi, SNR_{bitdB_{pp}}, 64\right)$$

$$y3_{pp} := P_{sym}\left(\frac{1}{180}\cdot\pi, SNR_{bitdB_{pp}}, 64\right)$$

$$y4_{pp} := P_{sym}\left(\frac{1.5}{180}\cdot\pi, SNR_{bitdB_{pp}}, 64\right)$$

$$y5_{pp} := P_{sym}\left(\frac{2.0}{180}\cdot\pi, SNR_{bitdB_{pp}}, 64\right)$$



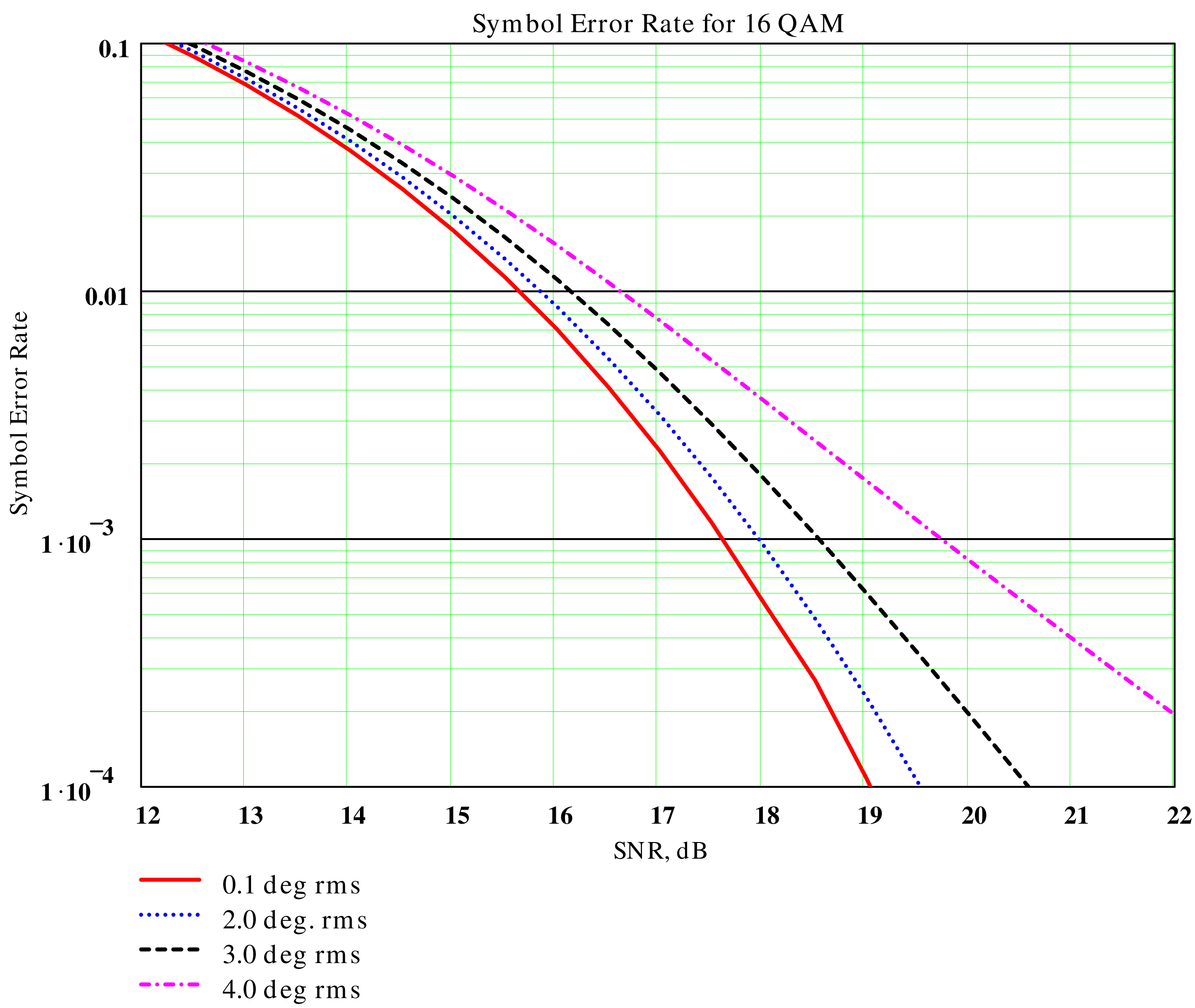
Now, assume the Tikhonov PDF for the phase error: 16 QAM Case

$$p_\theta(\theta, \sigma_\theta) := \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma_\theta} \cdot e^{\left(\frac{\cos(\theta) - 1}{\sigma_\theta^2}\right)}$$

$$PsWithNoise16(SNR_{bitdB}, \theta_{degrms}) := \int_{-0.5}^{0.5} p_\theta\left(\theta, \frac{\theta_{degrms}}{180} \cdot \pi\right) \cdot P_{sym}(\theta, SNR_{bitdB}, 16)\, d\theta$$

$$ph1_{pp} := PsWithNoise16\left(SNR_{bitdB_{pp}}, 0.1\right) \qquad ph2_{pp} := PsWithNoise16\left(SNR_{bitdB_{pp}}, 2\right)$$

$$ph3_{pp} := PsWithNoise16\left(SNR_{bitdB_{pp}}, 3\right) \qquad ph4_{pp} := PsWithNoise16\left(SNR_{bitdB_{pp}}, 4\right)$$



Now, assume the Tikhonov PDF for the phase error: 64 QAM Case

$$PsWithNoise64\left(SNR_{bitdB}, \theta_{degrms}\right) := \int_{-0.5}^{0.5} p\theta\left(\theta, \frac{\theta_{degrms}}{180} \cdot \pi\right) \cdot P_{sym}\left(\theta, SNR_{bitdB}, 64\right) d\theta$$

$$ph1_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 0.1\right)$$

$$ph2_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 1.0\right)$$

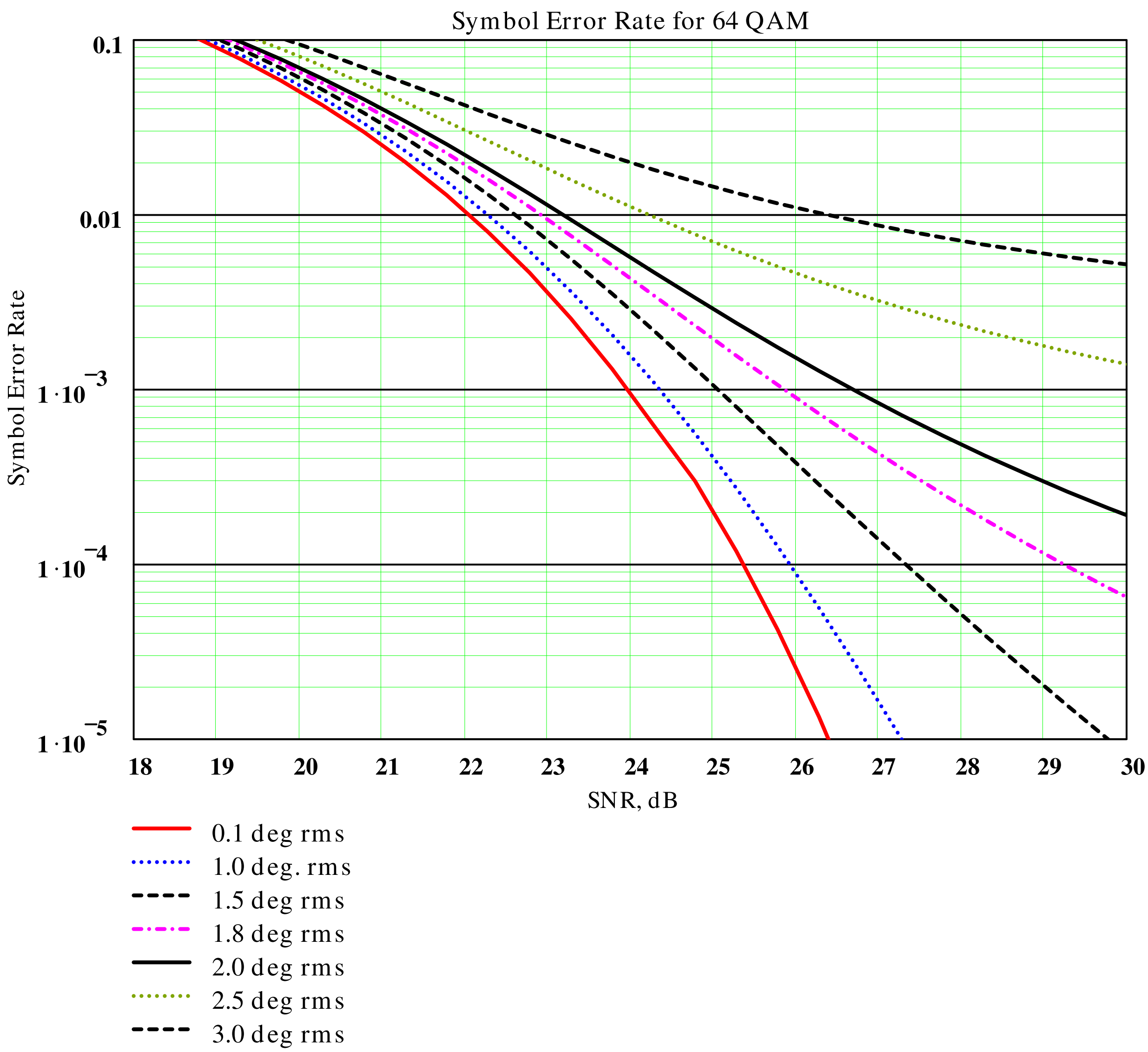
$$ph3_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 1.5\right)$$

$$ph4_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 1.8\right)$$

$$ph5_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 2\right)$$

$$ph6_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 2.5\right)$$

$$ph7_{pp} := PsWithNoise64\left(SNR_{bitdB_{pp}}, 3\right)$$



Now, assume the Tikhonov PDF for the phase error: 256 QAM Case

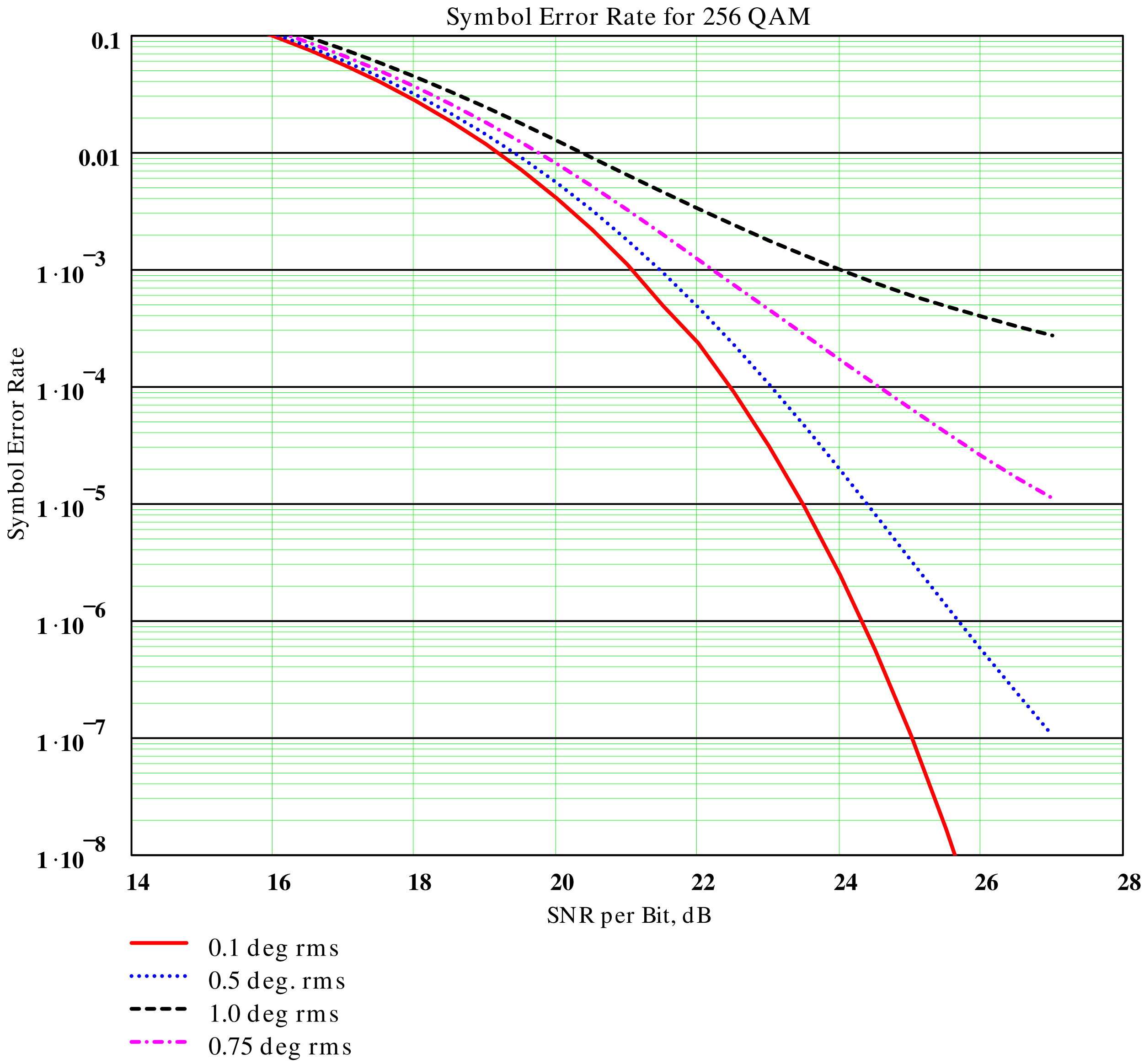
$$PsWithNoise256(SNR_{bitdB}, \theta_{degrms}) := \int_{-0.5}^{0.5} p\theta\left(\theta, \frac{\theta_{degrms}}{180} \cdot \pi\right) \cdot P_{sym}(\theta, SNR_{bitdB}, 256)\, d\theta$$

$$ph1_{pp} := PsWithNoise256\left(SNR_{bitdB_{pp}}, 0.1\right)$$

$$ph2_{pp} := PsWithNoise256\left(SNR_{bitdB_{pp}}, 0.5\right)$$

$$ph3_{pp} := PsWithNoise256\left(SNR_{bitdB_{pp}}, 1\right)$$

$$ph4_{pp} := PsWithNoise256\left(SNR_{bitdB_{pp}}, 0.75\right)$$



$$\mathrm{erf}(1) = 0.8427$$

$$\int_{-1}^{1} \frac{e^{\left(-u^2\right)}}{\sqrt{\pi}}\,du = 0.8427$$

Rayleigh Fading Compensation Method for 16QAM in Digital Land Mobile Radio Channels", IEEE 1989, S. Sampei, T. Sunaga

BER for Gray-coded 16-QAM with coherent detection

$$P_b(\gamma_o) := \frac{3}{8}\cdot \mathrm{erfc}\left(\sqrt{0.4\cdot\gamma_o}\right) - \frac{9}{64}\cdot \mathrm{erfc}\left(\sqrt{0.4\cdot\gamma_o}\right)^2$$

Rayleigh Fading Compensation for QAM in Land Mobile Radio Communications", IEE Trans. Vehicular Tech., May 1993, S. Sampei, T. Sunaga

BER for 64-QAM

$$P_b(\gamma_o) := \frac{4}{27}\cdot \mathrm{erfc}\left(\frac{1}{7}\cdot\gamma_o\right) - \frac{49}{384}\cdot \mathrm{erfc}\left(\sqrt{\frac{1}{7}\cdot\gamma_o}\right)^2$$

BER for 256 QAM

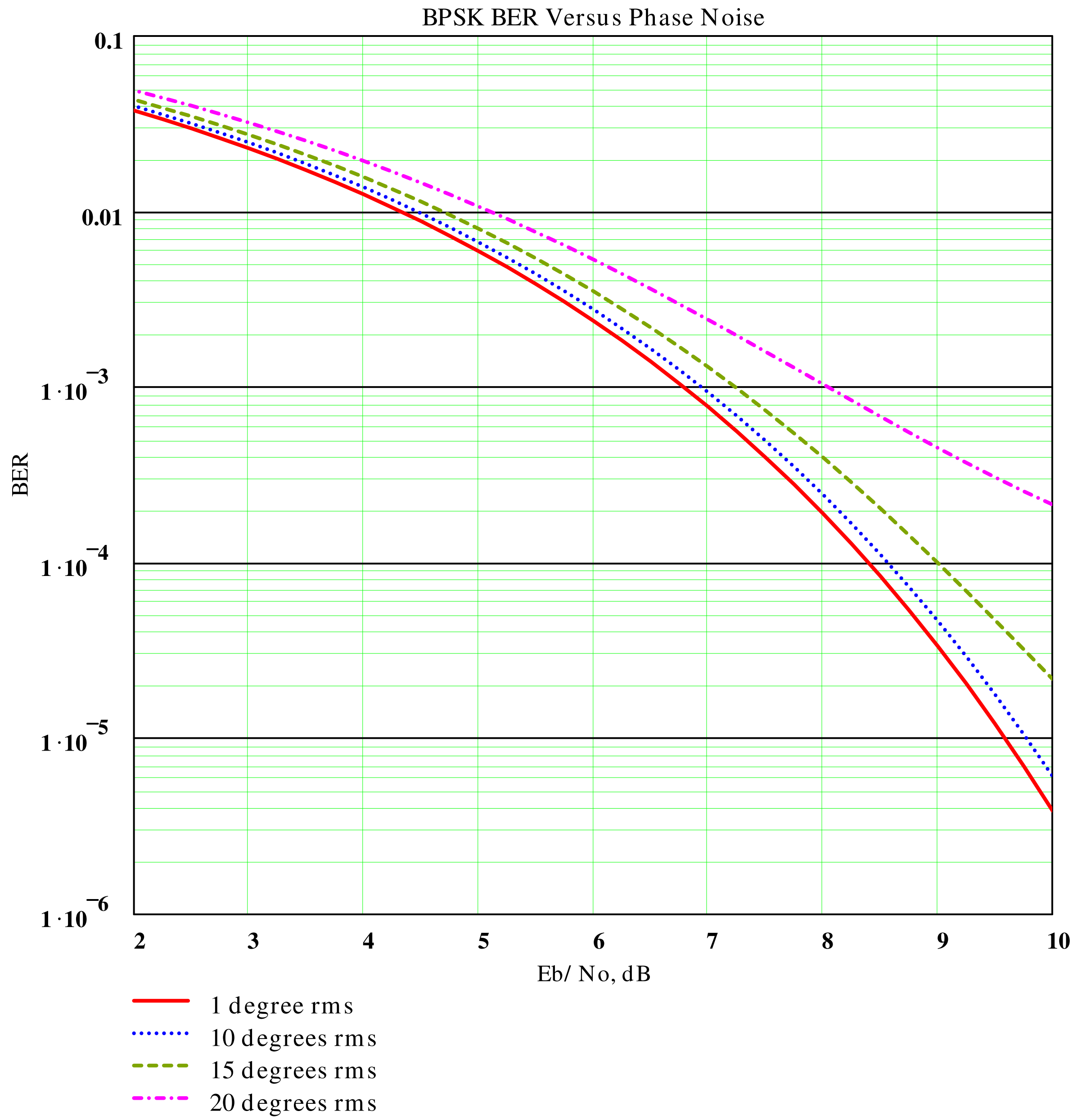
$$P_b(\gamma_o) := \frac{15}{64}\cdot \mathrm{erfc}\left(\sqrt{\frac{4}{85}\cdot\gamma_o}\right) - \frac{225}{2048}\cdot \mathrm{erfc}\left(\sqrt{\frac{4}{85}\cdot\gamma_o}\right)^2$$

Look at BER with BPSK

$kp := 0..40 \qquad x_{kp} := kp \cdot 0.25$

$$Pb(EbNo_{dB}, \sigma p) := 2 \cdot \int_0^{8 \cdot \sigma p} \frac{1}{2} \cdot \mathrm{erfc}\left(10^{0.05 \cdot EbNo_{dB}} \cdot \cos(\theta)\right) \cdot \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma p} \cdot e^{-0.5 \cdot \left(\frac{\theta}{\sigma p}\right)^2} \, d\theta$$

$y1_{kp} := Pb\left(x_{kp}, \frac{1}{180} \cdot \pi\right) \qquad y2_{kp} := Pb\left(x_{kp}, \frac{10}{180} \cdot \pi\right) \qquad y3_{kp} := Pb\left(x_{kp}, \frac{15}{180} \cdot \pi\right) \qquad y4_{kp} := Pb\left(x_{kp}, \frac{20}{180} \cdot \pi\right)$



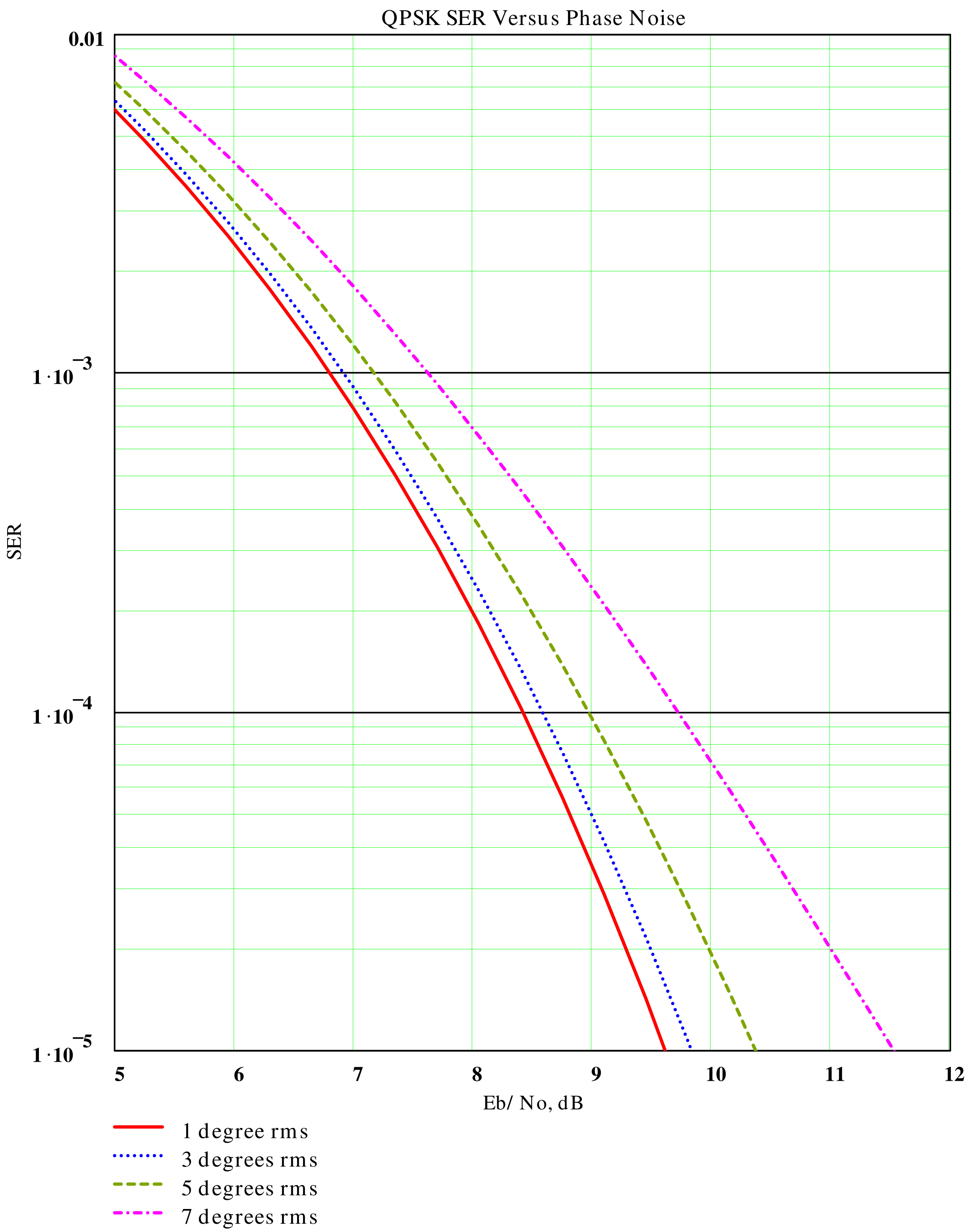
Look at BER with QPSK

$kp := 0..40 \qquad x_{kp} := kp \cdot 0.35$

$$Pbq(EbNo_{dB}, \sigma p) := \int_{0}^{8\cdot\sigma p} \frac{1}{2}\cdot \mathrm{erfc}\left[10^{0.05\cdot EbNo_{dB}}\cdot(\cos(\theta) - \sin(\theta))\right]\cdot\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma p}\cdot e^{-0.5\cdot\left(\frac{\theta}{\sigma p}\right)^2}\,d\theta\ ...$$

$$+ \int_{0}^{8\cdot\sigma p} \frac{1}{2}\cdot \mathrm{erfc}\left[10^{0.05\cdot EbNo_{dB}}\cdot(\cos(\theta) + \sin(\theta))\right]\cdot\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma p}\cdot e^{-0.5\cdot\left(\frac{\theta}{\sigma p}\right)^2}\,d\theta$$

$y1_{kp} := Pbq\left(x_{kp}, \frac{1}{180}\cdot\pi\right) \quad y2_{kp} := Pbq\left(x_{kp}, \frac{3}{180}\cdot\pi\right) \quad y3_{kp} := Pbq\left(x_{kp}, \frac{5}{180}\cdot\pi\right) \quad y4_{kp} := Pbq\left(x_{kp}, \frac{7}{180}\cdot\pi\right)$


QPSK SER Versus Phase Noise
0.01
$1 \cdot 10^{-3}$
$1 \cdot 10^{-4}$
$1 \cdot 10^{-5}$
SER
5
6
7
8
9
10
11
12
Eb/ No, dB
1 degree rms
3 degrees rms
5 degrees rms
7 degrees rms