Worksheet Calculations for Comm Design Paper

By: James A. Crawford
March, 2004

Continuous and Sampled PLLs (Type-2 3rd Order)

$\omega_n := 2\cdot\pi\cdot 12500$ $\quad K_d := \dfrac{0.001}{2\cdot\pi}$ $\quad F_s := 100\cdot 10^3$ $\quad jx := \sqrt{-1}$

$\zeta := 0.90$ $\quad K_v := 2\cdot\pi\cdot 10^7$

$N := 3000$

$$Cx := \left(\frac{\omega_n^{\,2}\cdot N}{K_d\cdot K_v}\right)^{-1} \qquad Cx = 5.404\times 10^{-10}$$

$C_1 := 0.1\cdot Cx$ $\qquad C_2 := 0.9\cdot Cx$

$$\tau_2 := \frac{2\cdot\zeta}{\omega_n} \qquad \tau_2 = 2.292\times 10^{-5} \qquad R_2 := \tau_2\cdot C_2^{-1} \qquad R_2 = 4.712\times 10^{4}$$

$$\tau_p := C_1\cdot C_2\cdot R_2\cdot\left(C_1 + C_2\right)^{-1}$$

$$G_{OL1}(s) := \left(\frac{\omega_n}{s}\right)^2\cdot\left(\frac{1 + s\cdot\tau_2}{1 + s\cdot\tau_p}\right)$$

$$G_{OL2}(s) := \frac{\omega_n^{\,2}}{s}\cdot\left(\tau_2 - \tau_p\right) + \left(\frac{\omega_n}{s}\right)^2 + \frac{\omega_n^{\,2}}{1 + s\cdot\tau_p}\cdot\left(\tau_p^{\,2} - \tau_2\cdot\tau_p\right)$$

Partial fraction form

Numerical Check that both open-loop gain functions provide same result.

$G_{OL1}(jx\cdot 4456.0) = -310.954 - 28.551i$ $\qquad G_{OL2}(jx\cdot 4456.0) = -310.954 - 28.551i$

$T := F_s^{-1}$

$$G_{OL}(z) := \omega_n^{\,2}\cdot\left[\left(\tau_2 - \tau_p\right)\cdot\frac{z}{z-1} + \frac{z\cdot T}{(z-1)^2} + \frac{\left(\tau_p - \tau_2\right)\cdot z}{z - e^{\frac{-T}{\tau_p}}}\right]$$

Gol(z) in easily invertable form

Look at open-loop gain functions with and without sampling effects included.

$N_{pts} := 50 \qquad pp := 0 .. N_{pts} - 1$

$$fsw_{pp} := \frac{1.001}{2 \cdot \pi} \cdot 10^{6.19 \cdot \left(\frac{pp}{N_{pts}}\right)}$$

$$Ms_{pp} := 10 \cdot \log\left[\left(\left|G_{OL2}\left(jx \cdot 2 \cdot \pi \cdot fsw_{pp}\right)\right|\right)^2\right] \qquad Mz_{pp} := 10 \cdot \log\left[\left(\left|T \cdot G_{OL}\left(e^{jx \cdot 2 \cdot \pi \cdot fsw_{pp} \cdot T}\right)\right|\right)^2\right]$$

$$Gsum(s, nm) := 10 \cdot \log\left[\left(\left|\sum_{kk=-nm}^{nm} G_{OL1}\left(s + \frac{jx \cdot kk \cdot 2 \cdot \pi}{T}\right)\right|\right)^2\right]$$

$G1_{pp} := Gsum\left(jx \cdot 2 \cdot \pi \cdot fsw_{pp}, 1\right)$

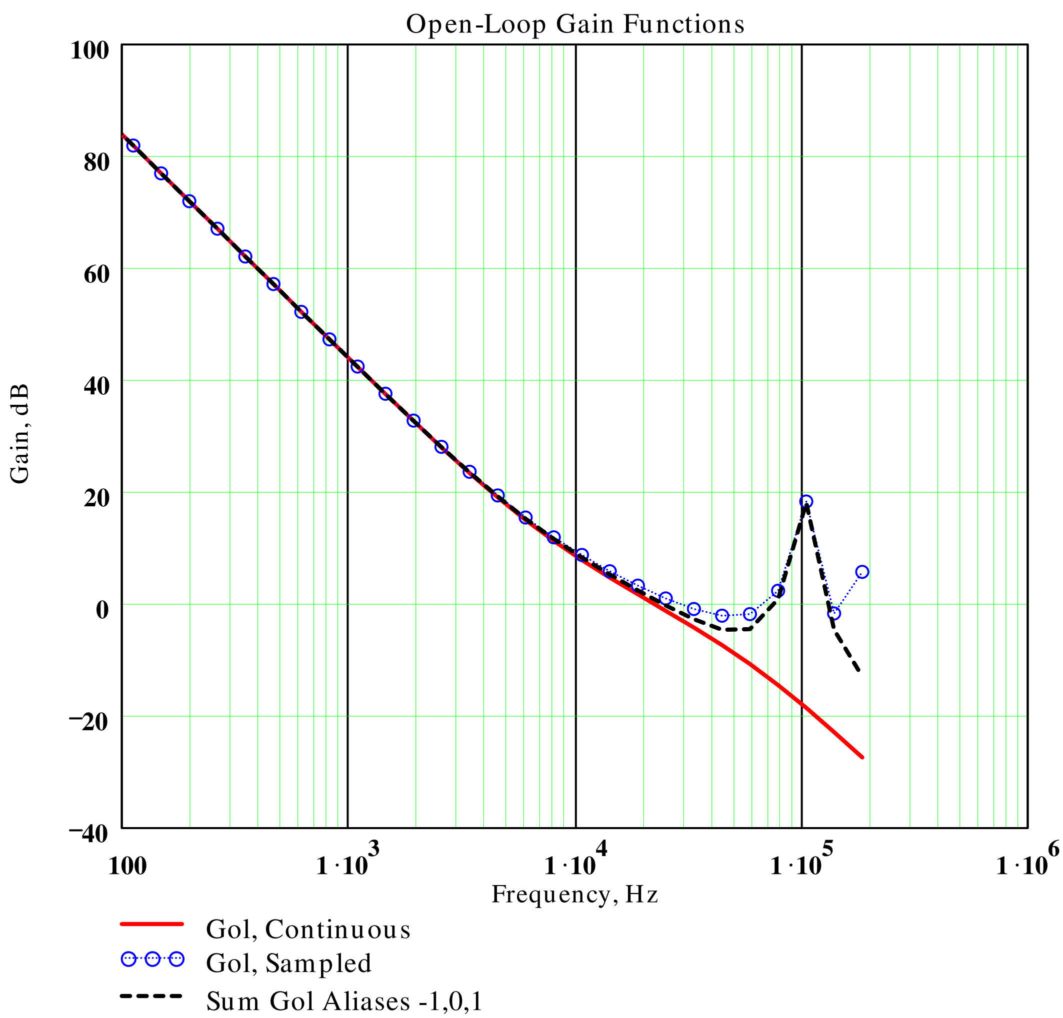
$G2_{pp} := Gsum\left(jx \cdot 2 \cdot \pi \cdot fsw_{pp}, 2\right)$

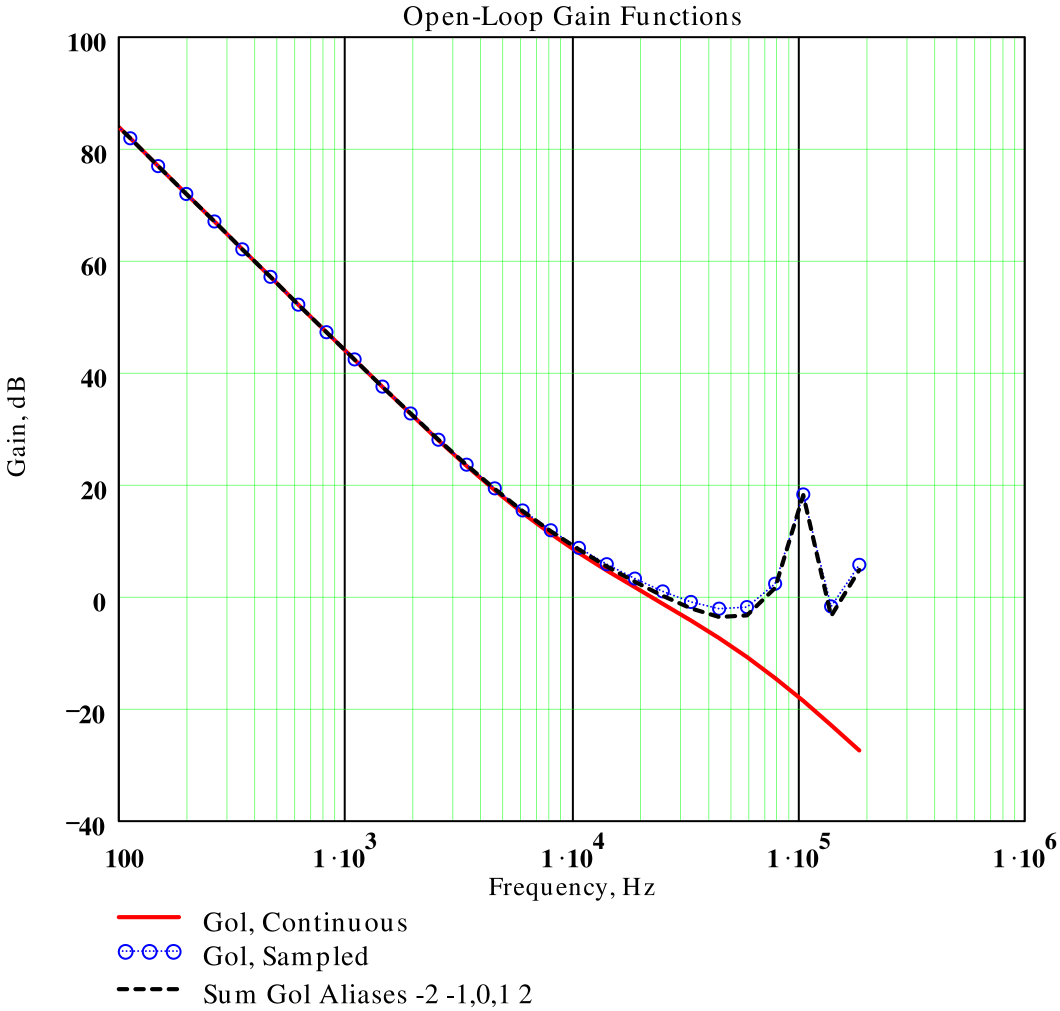
$G5_{pp} := Gsum\left(jx \cdot 2 \cdot \pi \cdot fsw_{pp}, 5\right)$

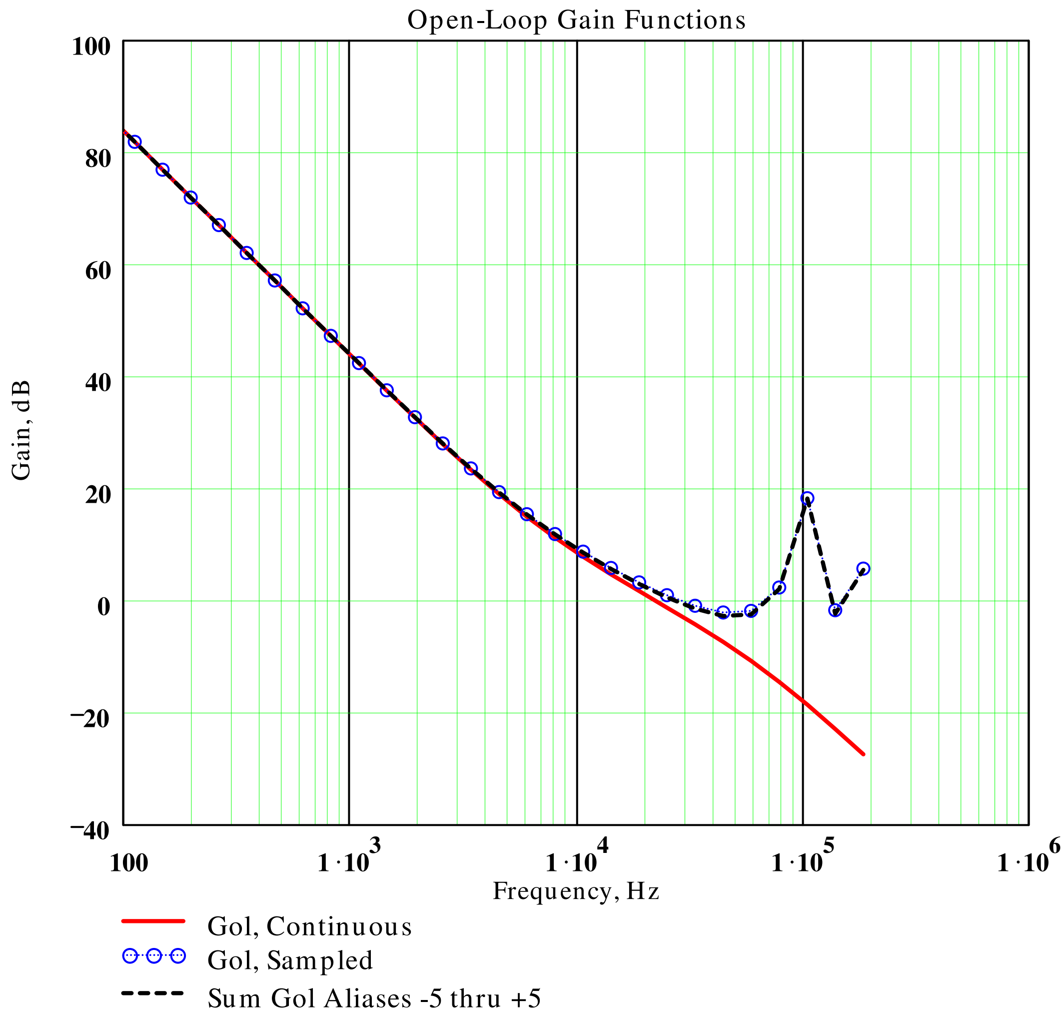
$$G_{CL}(s) := 10 \cdot \log\left[\left(\left|\frac{G_{OL1}(s)}{1 + T \cdot G_{OL}\left(e^{s \cdot T}\right)}\right|\right)^2\right]$$

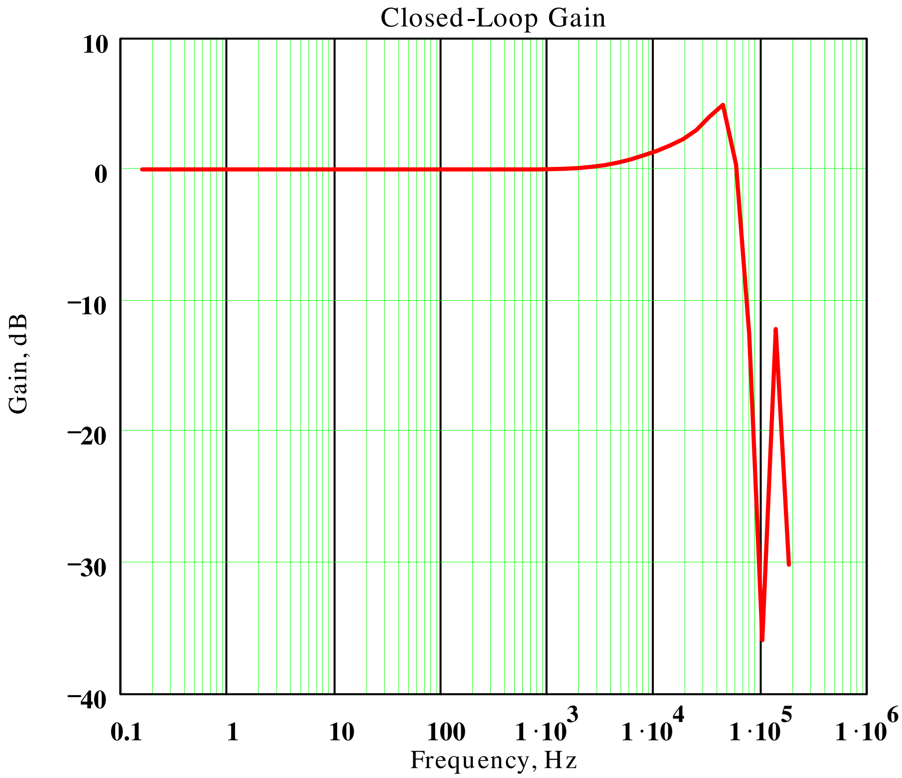
$H1_{pp} := G_{CL}\left(jx \cdot 2 \cdot \pi \cdot fsw_{pp}\right)$

Look at the impact of including more or less aliasing terms in Poisson Sum approximation for Gol(z)




Open-Loop Gain Functions
100
80
60
40
20
0
−20
−40
Gain, dB
100
1·10^3
1·10^4
1·10^5
1·10^6
Frequency, Hz
Gol, Continuous
Gol, Sampled
Sum Gol Aliases -2 -1,0,1 2


Open-Loop Gain Functions
Gain, dB
100
80
60
40
20
0
−20
−40
100
1·10^3
1·10^4
1·10^5
1·10^6
Frequency, Hz
Gol, Continuous
Gol, Sampled
Sum Gol Aliases -5 thru +5


Closed-Loop Gain
10
0
−10
−20
−30
−40
Gain, dB
0.1
1
10
100
1·10^3
1·10^4
1·10^5
1·10^6
Frequency, Hz

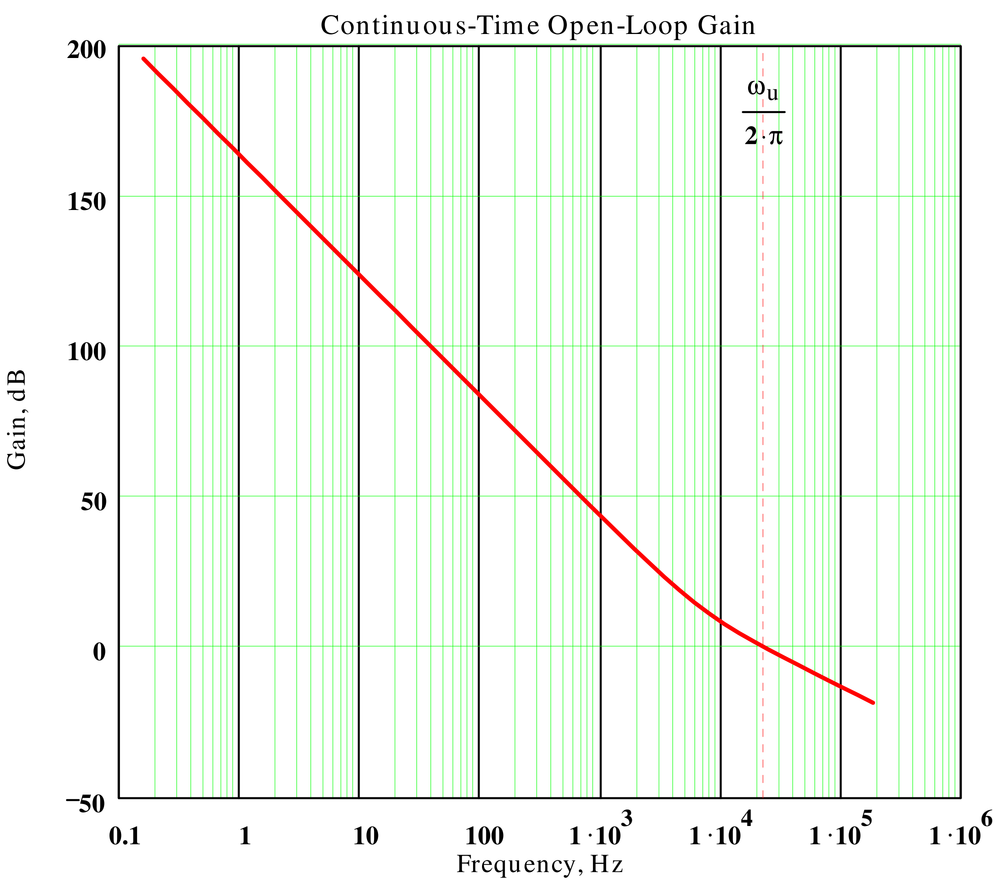
Confirm Unity-Gain Open-Loop Frequency for Classic Type-2 PLL (Continuous-Time)

$$G_{OLC}(s,\omega_n,\zeta) := \left(\frac{\omega_n}{s}\right)^2 \cdot \left(1 + \frac{2\cdot\zeta\cdot s}{\omega_n}\right) \qquad \omega_n = 7.854 \times 10^4$$

$$\omega_{unity}(\omega_n,\zeta) := \omega_n \cdot \sqrt{2\cdot\zeta^2 + \sqrt{4\cdot\zeta^4 + 1}} \qquad \zeta = 0.9$$

$$\omega_u := \omega_{unity}(\omega_n,\zeta) \qquad \omega_u = 1.474 \times 10^5$$

$$py_{pp} := 10\cdot\log\left[\left(\left|G_{OLC}(jx\cdot 2\cdot\pi\cdot fsw_{pp},\omega_n,\zeta)\right|\right)^2\right]$$



Where does unity-gain closed-loop occur in frequency?

$$\omega_n = 7.854 \times 10^4 \qquad \zeta = 0.9$$

$$H(s, \omega_n, \zeta) := 10 \cdot \log\left[\left(\left|\frac{G_{OLC}(s, \omega_n, \zeta)}{1 + G_{OLC}(s, \omega_n, \zeta)}\right|\right)^2\right]$$

$$H\left(jx \cdot \sqrt{2} \cdot \omega_n, \omega_n, \zeta\right) = 0$$

$$Hmag_{pp} := H\left(jx \cdot 2 \cdot \pi \cdot fsw_{pp}, \omega_n, \zeta\right)$$

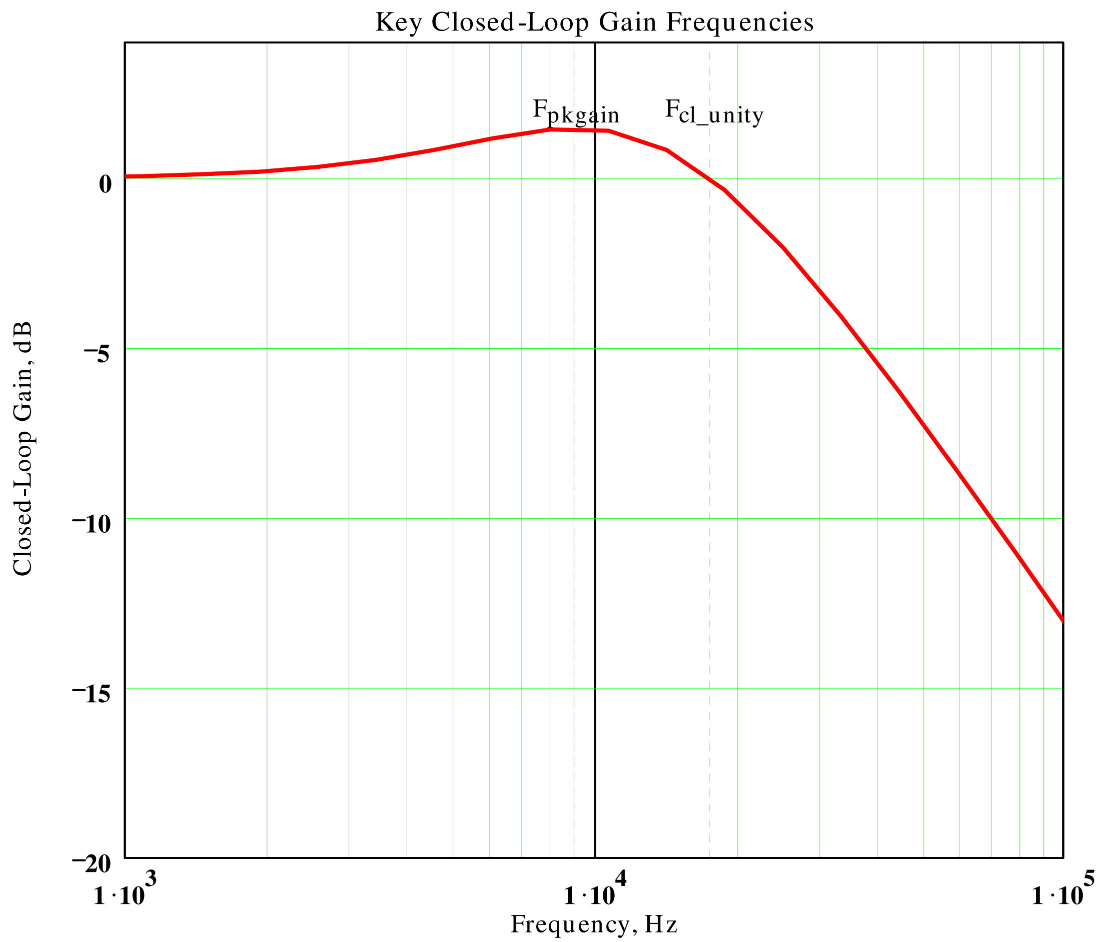
$$F_{cl\_unity} := \frac{\sqrt{2} \cdot \omega_n}{2 \cdot \pi} \qquad F_{cl\_unity} = 1.768 \times 10^4$$

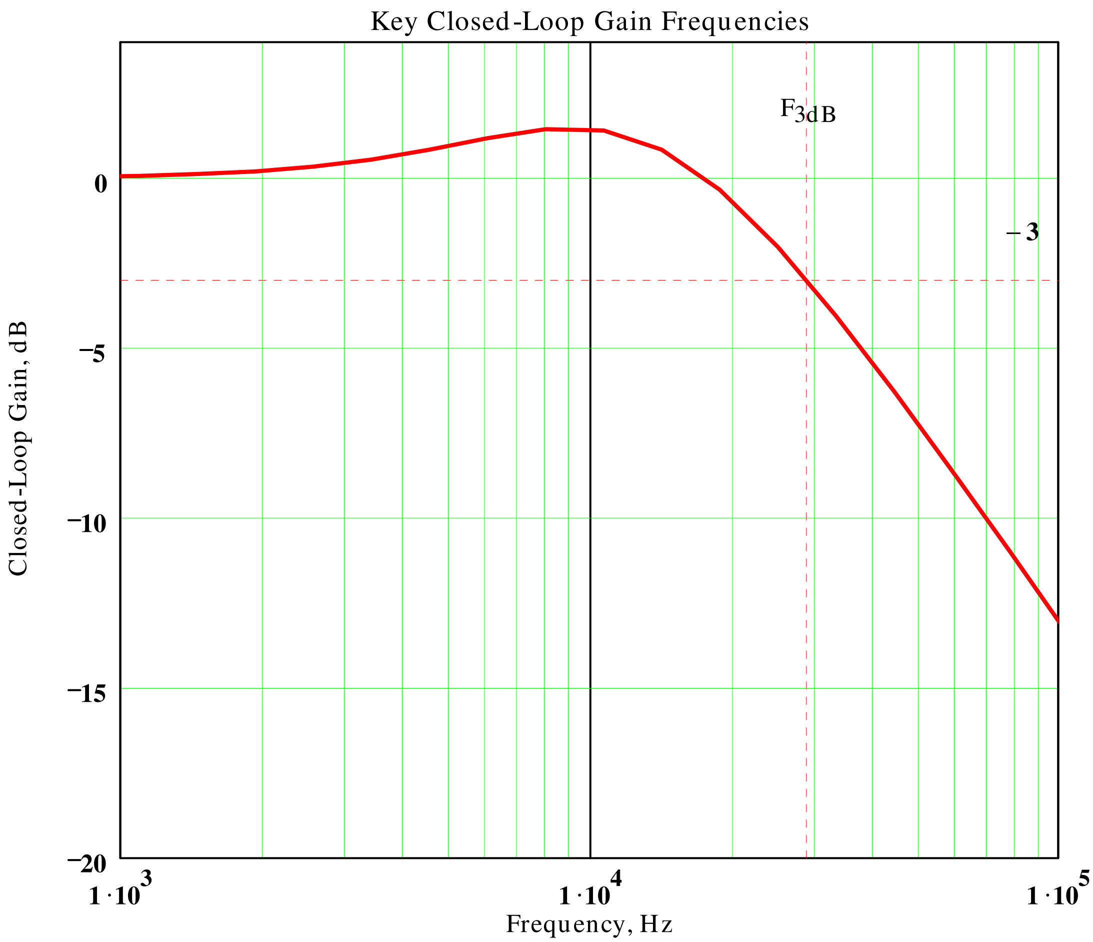
Frequency for Closed-Loop Peak Gain?

$$F_{pkgain} := \frac{\omega_n}{2 \cdot \pi} \cdot \frac{1}{2 \cdot \zeta} \cdot \sqrt{\sqrt{1 + 8 \cdot \zeta^2} - 1} \qquad F_{pkgain} = 9.147 \times 10^3$$

Frequency for Closed-Loop -3dB Gain?

$$F_{3dB} := \frac{\omega_n}{2 \cdot \pi} \cdot \sqrt{1 + 2 \cdot \zeta^2 + 2 \cdot \sqrt{\zeta^4 + \zeta^2 + \frac{1}{2}}} \qquad F_{3dB} = 2.911 \times 10^4$$

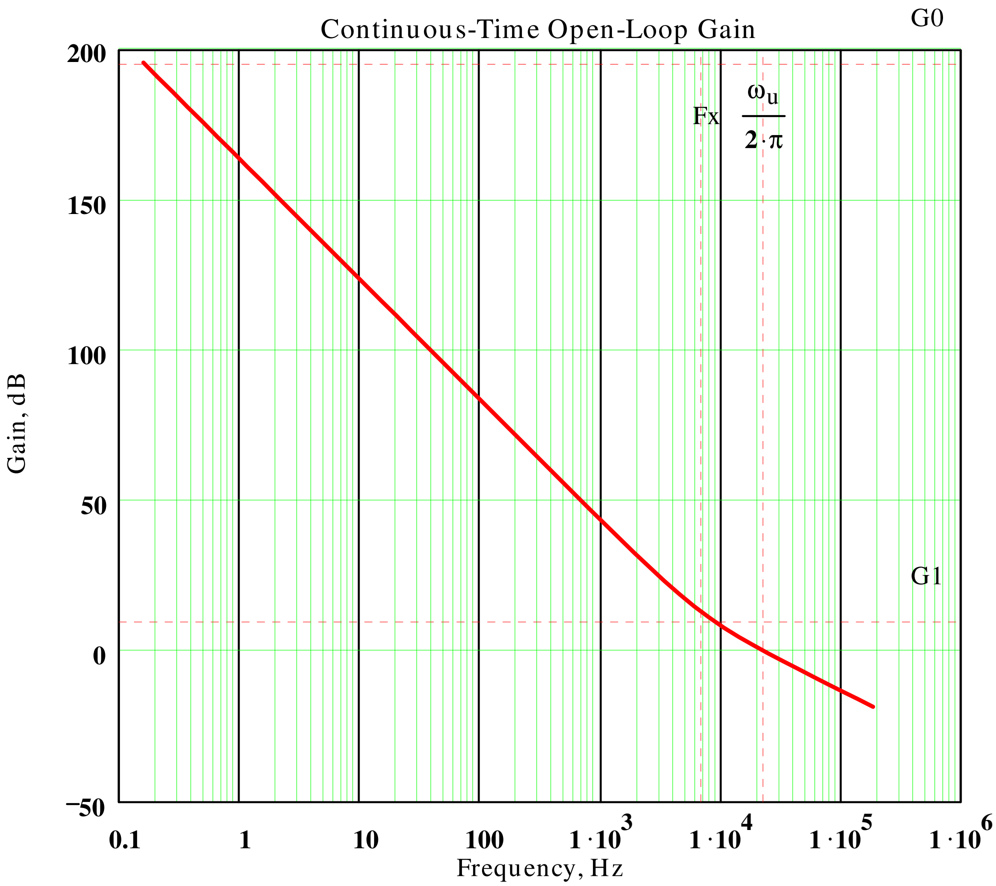

Key Closed-Loop Gain Frequencies
F_{pkgain}
F_{cl_unity}
0
−5
−10
−15
−20
Closed-Loop Gain, dB
$1 \cdot 10^3$
$1 \cdot 10^4$
$1 \cdot 10^5$
Frequency, Hz


Key Closed-Loop Gain Frequencies
F3dB
−3
0
−5
−10
−15
−20
Closed-Loop Gain, dB
1·10^3
1·10^4
1·10^5
Frequency, Hz

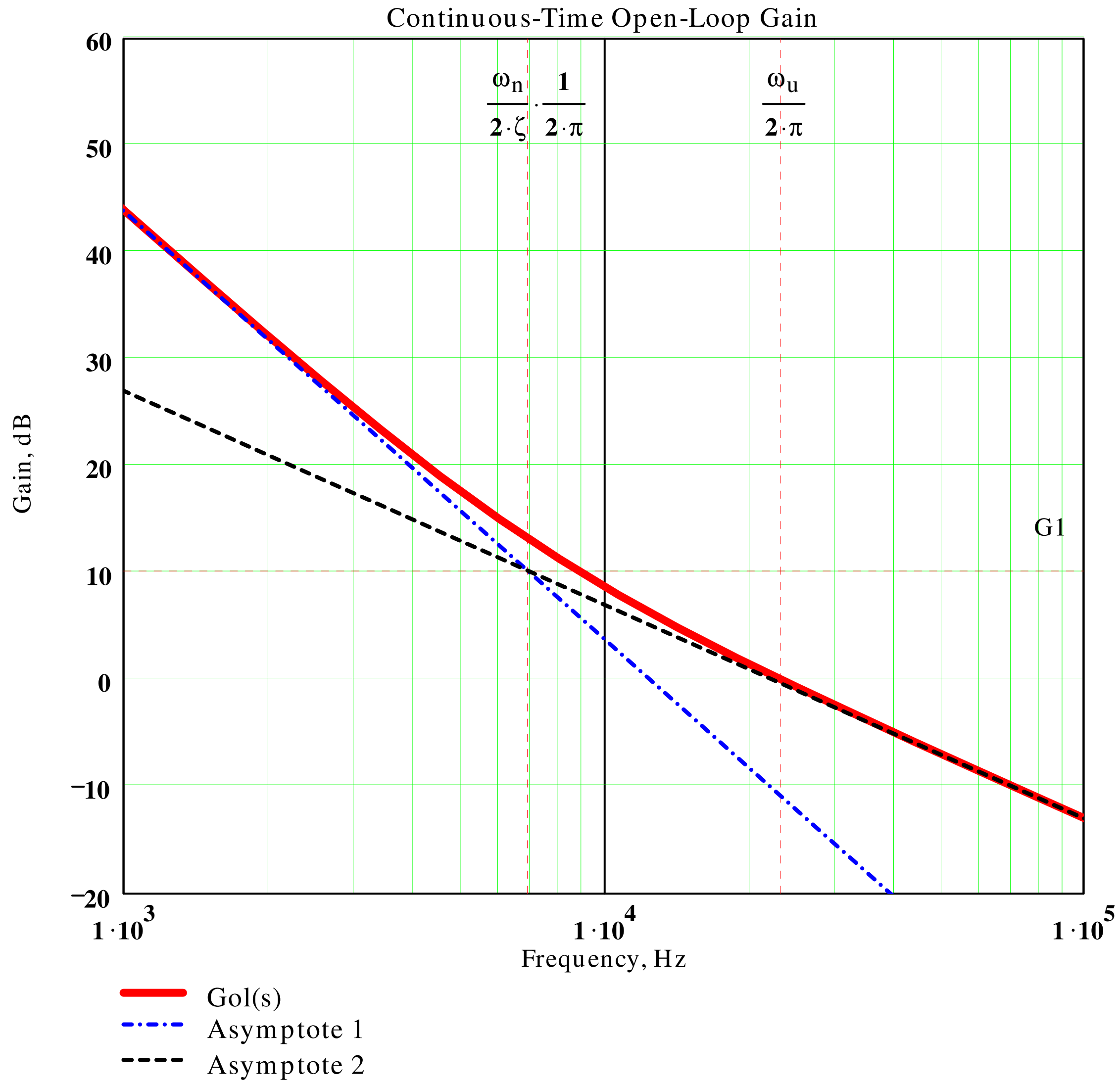
Convenient Approximations for Key Points of Open-Loop Gain Function

$$G0 := 10 \cdot \log\left(\omega_n^4 + 4 \cdot \omega_n^2 \cdot \zeta^2\right) \qquad Fx := \frac{\omega_n}{2 \cdot \zeta} \cdot \frac{1}{2 \cdot \pi} \qquad G0 = 195.804$$

$$G1 := 40 \cdot \log(2 \cdot \zeta) \qquad G1 = 10.211$$



$$SL1_{pp} := G0 - 40 \cdot \log\left(2 \cdot \pi \cdot fsw_{pp}\right) \qquad SL2_{pp} := 20 \cdot \log\left(2 \cdot \zeta \cdot \frac{\omega_n}{2 \cdot \pi \cdot fsw_{pp}}\right)$$



Closed-Loop Effect on VCO Phase Noise

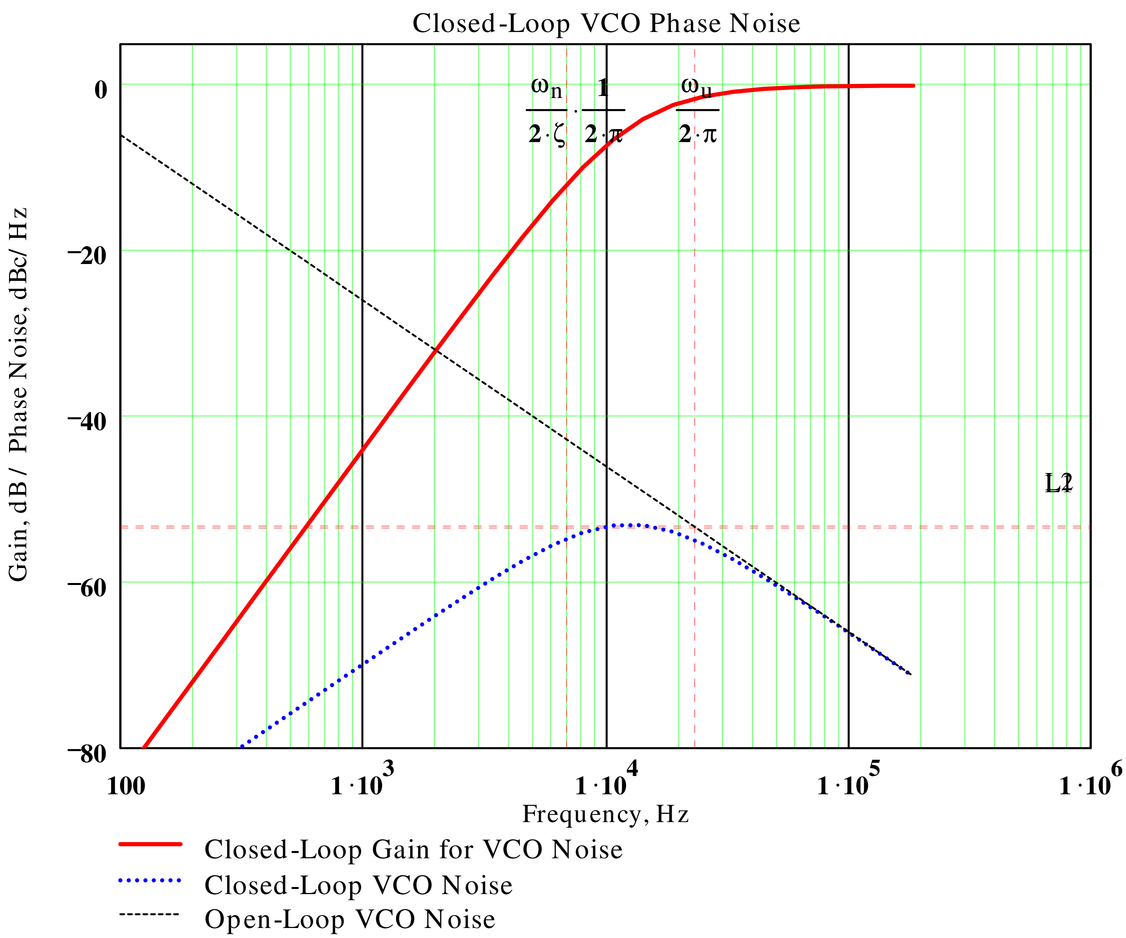
Transfer function for VCO Noise in Closed-Loop

$$pz_{pp} := 10\cdot\log\left[\left(\left|\frac{1}{1+G_{OLC}\left(jx\cdot 2\cdot\pi\cdot fsw_{pp},\omega_n,\zeta\right)}\right|\right)^2\right]$$

VCO Phase Noise Level at 1 rad/sec = L dBc/Hz $\quad L_{dBc} := 50$

$S_{vco}(f) := L_{dBc} - 20\cdot\log(2\cdot\pi\cdot f)$ $\quad pv_{pp} := S_{vco}(fsw_{pp})$ $\quad S_{vco\_cl_{pp}} := pv_{pp} + pz_{pp}$

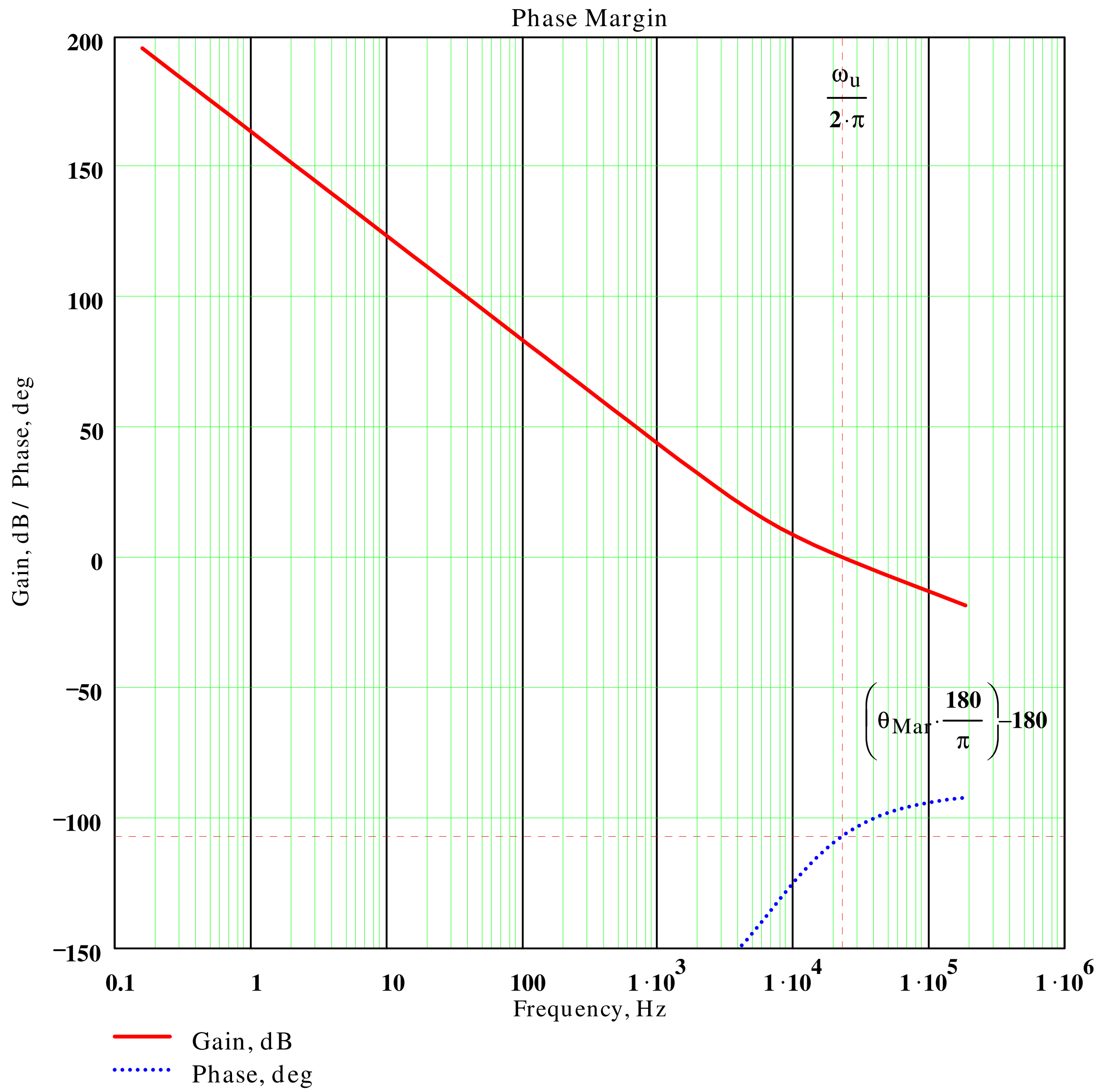
$L1 := L_{dBc} - 20\cdot\log(\omega_u)$ $\quad L2 := L_{dBc} - 20\cdot\log(2\cdot\zeta\cdot\omega_n)$



Phase Margin for Classic Time-Continuous Type-2 PLL

$\theta_{Mar} := \operatorname{atan}\left(2\cdot\zeta\cdot\sqrt{2\cdot\zeta^2+\sqrt{4\cdot\zeta^4+1}}\right)$ $\theta_{Mar} = 1.283$ $\theta_{Mar}\cdot 180\cdot\pi^{-1} = 73.514$

$Ang_{pp} := \arg\left(G_{OLC}\left(jx\cdot 2\cdot\pi\cdot fsw_{pp}, \omega_n, \zeta\right)\right)\cdot\frac{180}{\pi}$ $\arg\left(G_{OLC}\left(jx\cdot\omega_u, \omega_n, \zeta\right)\right) - \theta_{Mar} = -3.14159$



Transient Resposnes: Step-Frequency to Phase-Error Response

NOTE: All responses assume that feedback divider N= 1

$\omega_n = 7.854 \times 10^4$

$\zeta = 0.9$

$$\frac{\omega_n^2 \cdot \left(1 + \frac{2\cdot\zeta\cdot s}{\omega_n}\right)}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2} \cdot \frac{2\cdot\pi\,\Delta F}{s^2}$$

Laplace Transform Form

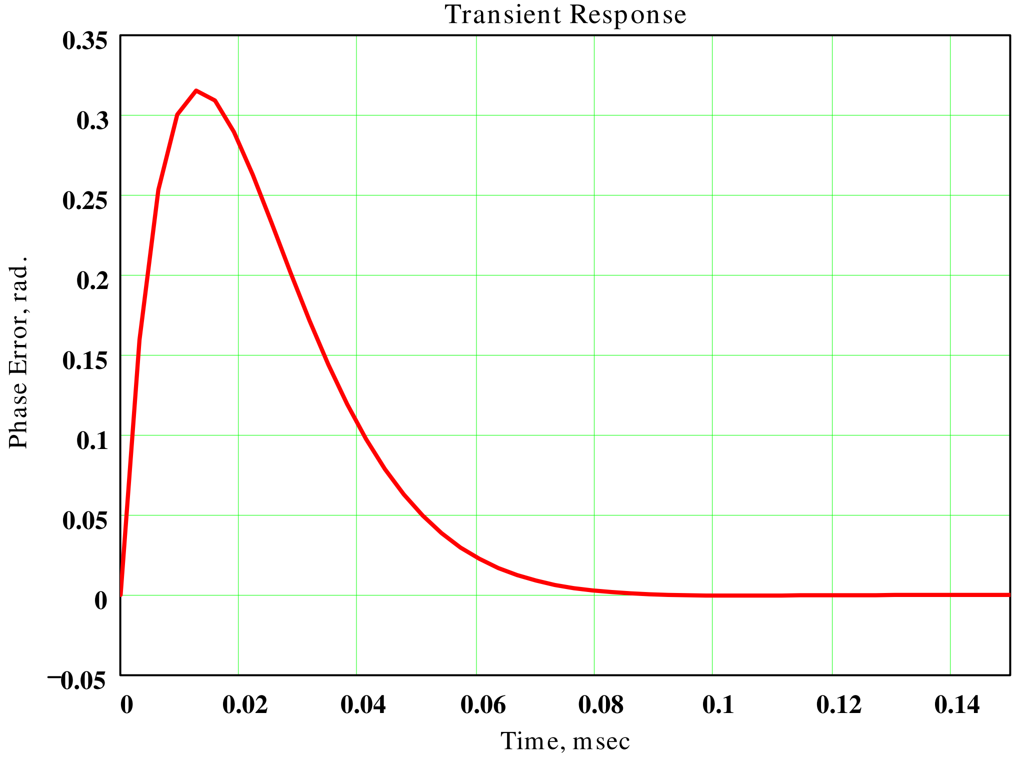
$\Delta F := 10000$

$$\theta_{pd}(t, \omega_n, \zeta) := \frac{2\cdot\pi\cdot\Delta F}{\omega_n} \cdot \frac{e^{-\zeta\cdot\omega_n\cdot t}}{\sqrt{1-\zeta^2}} \cdot \sin\left(\omega_n \cdot \sqrt{1-\zeta^2} \cdot t\right)$$

$dt := \frac{0.25}{\omega_n}$ $\quad nn := 0 .. 50$

$tm_{nn} := nn \cdot dt$

$pa1_{nn} := \theta_{pd}(tm_{nn}, \omega_n, \zeta)$



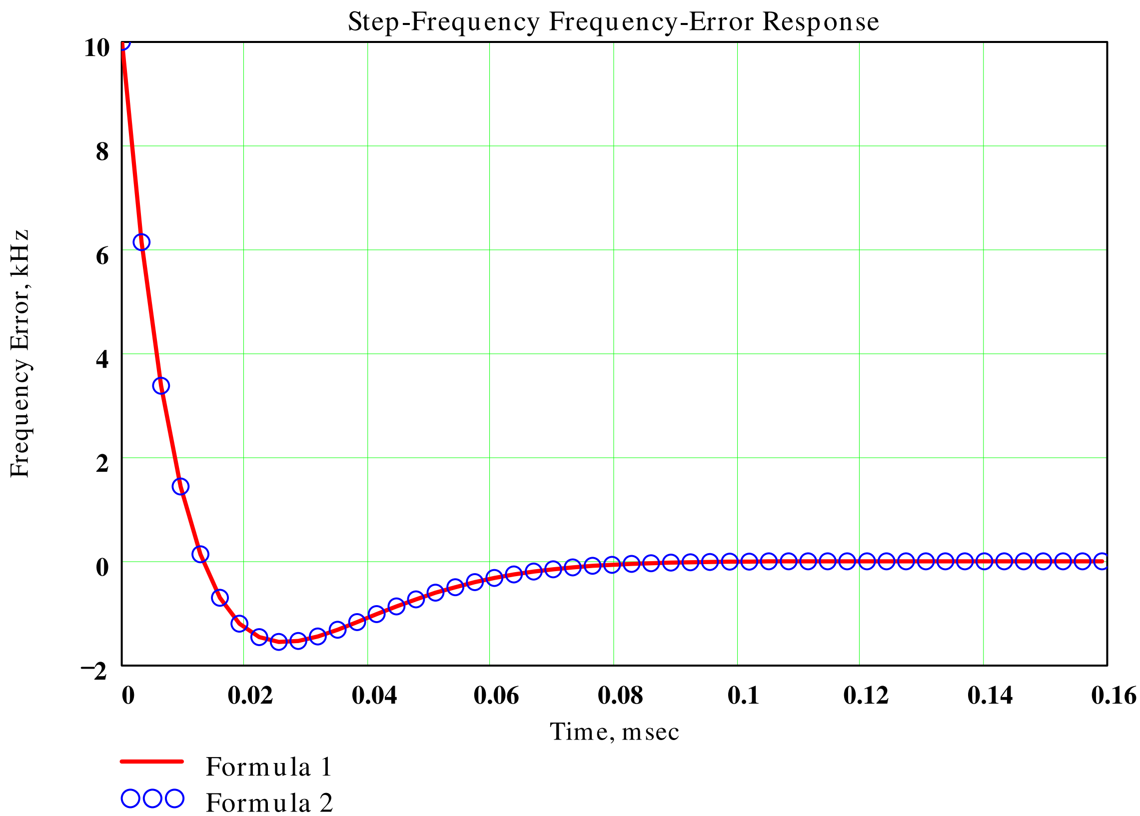
Transient Response: Step-Frequency to Frequency Error

$$\frac{2\cdot\pi\cdot\Delta F}{\omega_n}\cdot\frac{e^{-\zeta\cdot\omega_n\cdot t}}{\sqrt{1-\zeta^2}}\cdot\sin\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot t\right)$$ (time differentiate)

$$f_{pd1}(t,\omega_n,\zeta) := -\Delta F\cdot\zeta\cdot\frac{\exp(-\zeta\cdot\omega_n\cdot t)}{\left(1-\zeta^2\right)^{\left(\frac{1}{2}\right)}}\cdot\sin\left[\omega_n\cdot\left(1-\zeta^2\right)^{\left(\frac{1}{2}\right)}\cdot t\right]+\Delta F\cdot\exp(-\zeta\cdot\omega_n\cdot t)\cdot\cos\left[\omega_n\cdot\left(1-\zeta^2\right)^{\left(\frac{1}{2}\right)}\cdot t\right]$$

$$f_{pd11}(t,\omega_n,\zeta) := \Delta F\cdot e^{-\zeta\cdot\omega_n\cdot t}\cdot\left(\cos\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot t\right)-\frac{\zeta}{\sqrt{1-\zeta^2}}\cdot\sin\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot t\right)\right)$$

$pb1_{nn} := f_{pd1}(tm_{nn},\omega_n,\zeta)$ $pb2_{nn} := f_{pd11}(tm_{nn},\omega_n,\zeta)$



Transient Response: Step-Phase to Phase Error

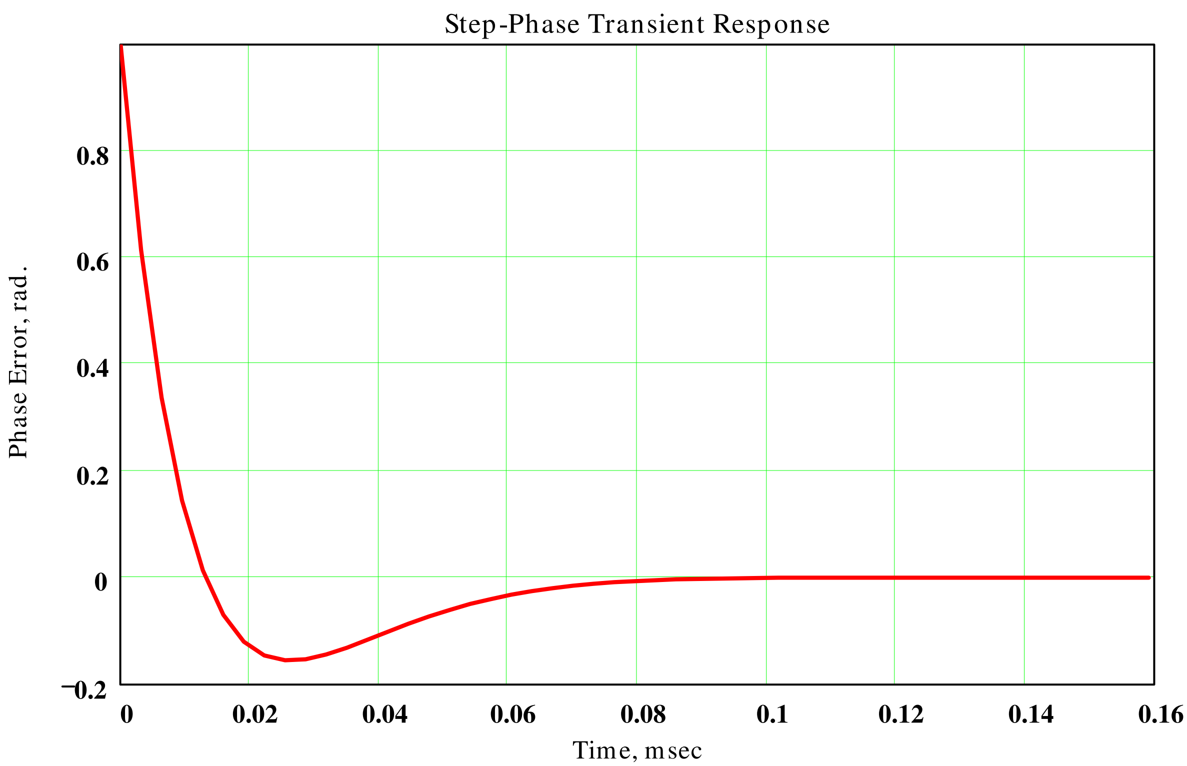
$\Delta\theta := 1$

$$\frac{\omega_n^{2}\cdot\left(1+\frac{2\cdot\zeta\cdot s}{\omega_n}\right)}{s^{2}+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{2}}\cdot\frac{\Delta\theta}{s}$$

Laplace Transform Form

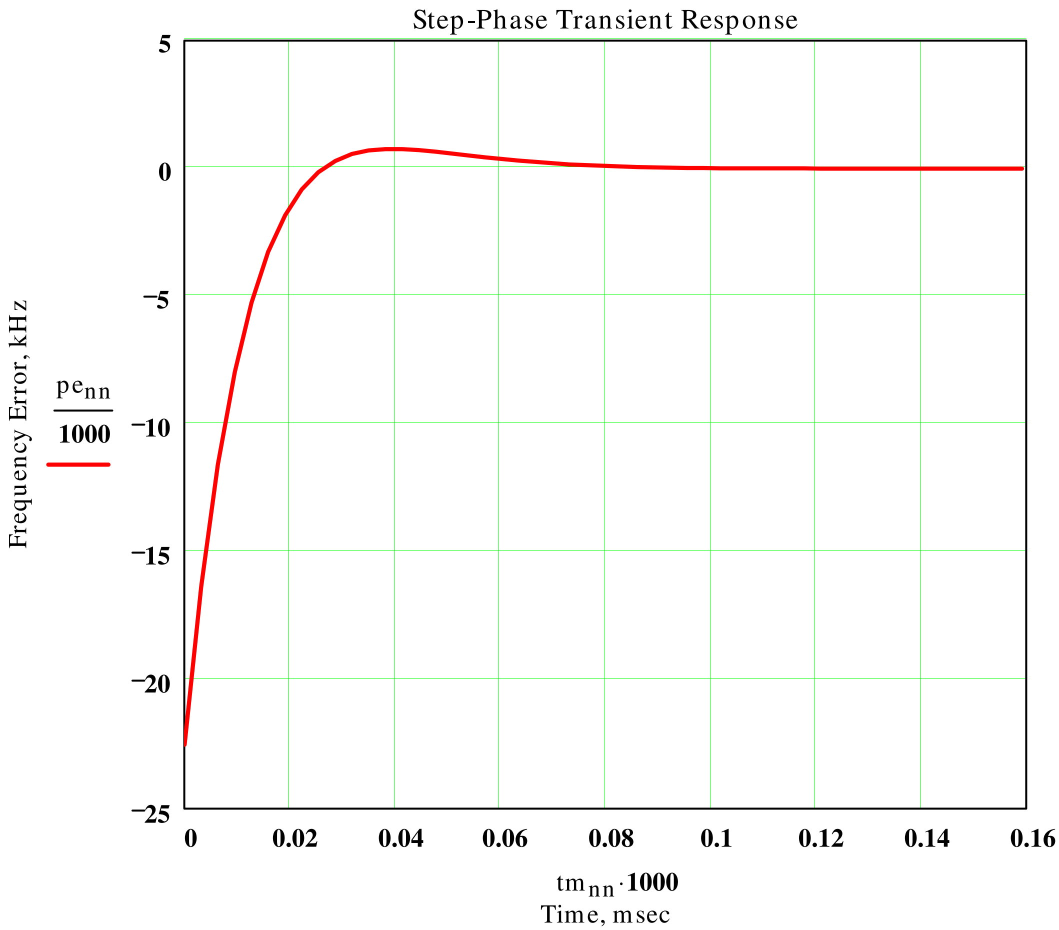
$$\theta_{pd2}(t,\omega_n,\zeta) := \Delta\theta\cdot e^{-\zeta\cdot\omega_n\cdot t}\cdot\left(\cos\left(\omega_n\cdot\sqrt{1-\zeta^{2}}\cdot t\right)-\frac{\zeta}{\sqrt{1-\zeta^{2}}}\cdot\sin\left(\omega_n\cdot\sqrt{1-\zeta^{2}}\cdot t\right)\right)$$

$$pc1_{nn} := \theta_{pd2}(tm_{nn},\omega_n,\zeta)$$



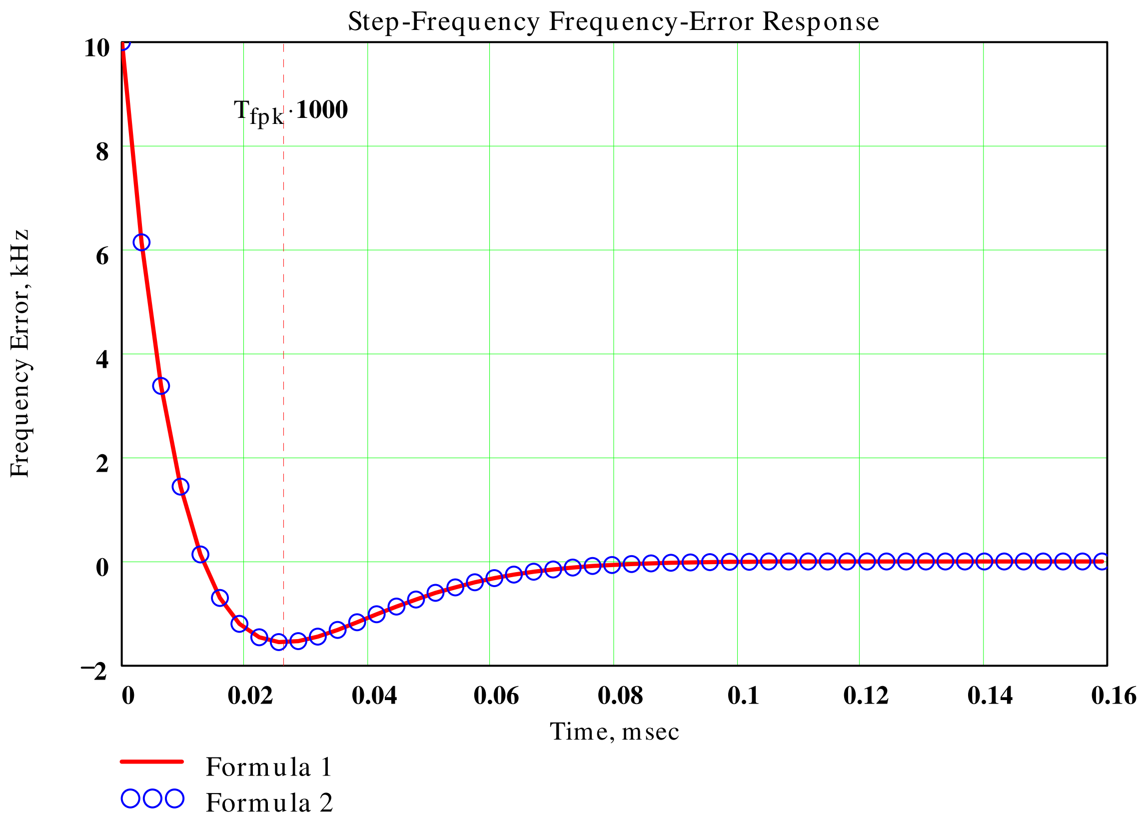
Transient Response: Step-Phase to Frequency Error

$$pe_{nn} := \frac{\Delta\theta}{2\cdot\pi}\cdot\omega_n\cdot e^{-\zeta\cdot\omega_n\cdot tm_{nn}}\cdot\left(\frac{2\cdot\zeta^2-1}{\sqrt{1-\zeta^2}}\cdot\sin\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot tm_{nn}\right)-2\cdot\zeta\cdot\cos\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot tm_{nn}\right)\right)$$



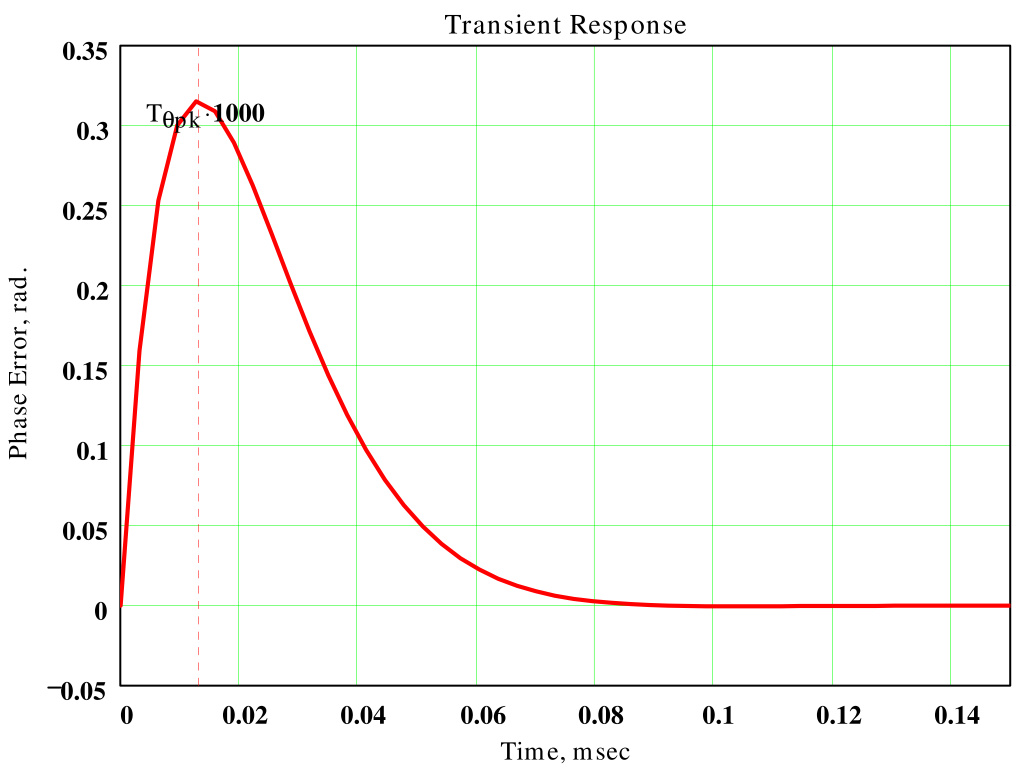
Peak Frequency Overshoot for Frequency-Step Input

$$T_{fpk} := \frac{2}{\omega_n \cdot \sqrt{1-\zeta^2}} \cdot \operatorname{atan}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$



Peak Phase Error for Frequency-Step Input

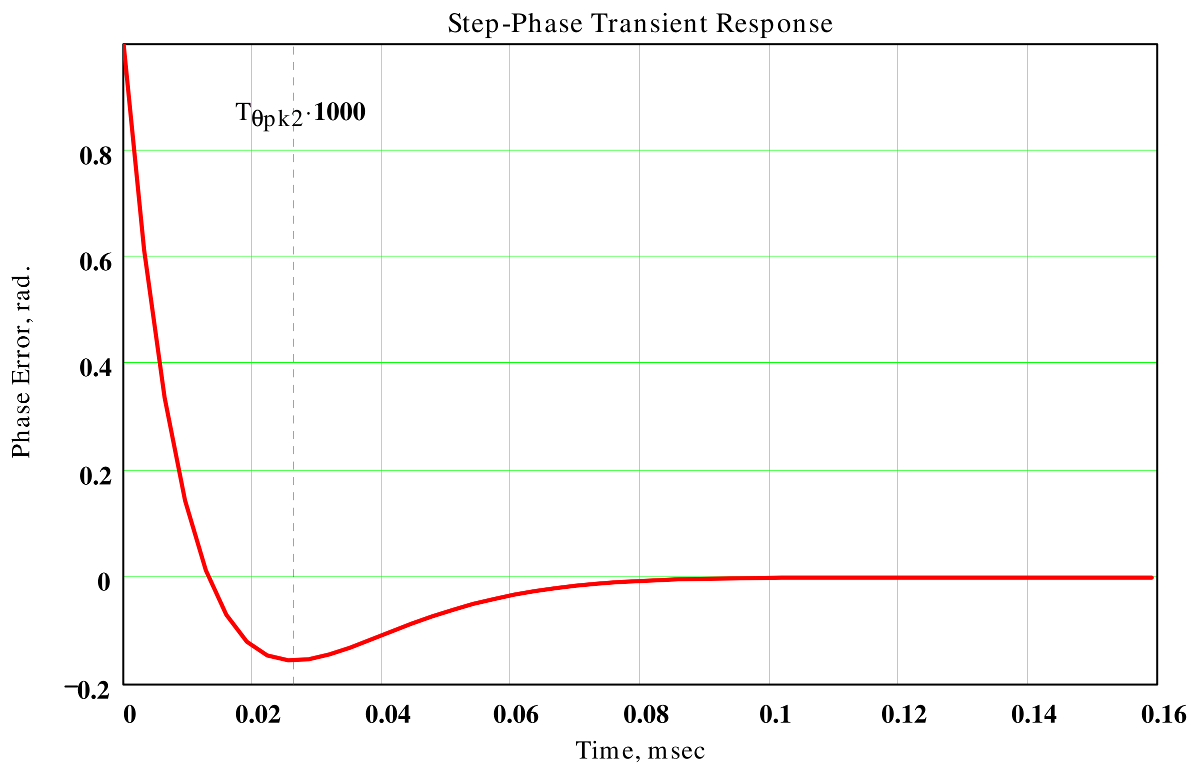
$$T_{\theta pk} := \frac{1}{\omega_n \cdot \sqrt{1-\zeta^2}} \operatorname{atan}\left[\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)^{-1}\right]$$



Peak Phase-Error for Step-Phase Input

$$T_{\theta pk2} := \frac{1}{\omega_n \cdot \sqrt{1-\zeta^2}} \cdot \mathrm{atan}\left(\frac{2 \cdot \zeta \cdot \sqrt{1-\zeta^2}}{2 \cdot \zeta^2 - 1}\right)$$

$$T_{\theta pk2} = 2.635 \times 10^{-5}$$



Sampled Type-2 System with Sample-Hold

Case 4 from My Book

$$F_s := 10^5 \qquad T_s := F_s^{-1}$$

$$\omega_n := 2\cdot\pi\cdot 0.020\cdot F_s \qquad K_d := \frac{1}{\pi} \qquad K_v := 2\cdot\pi\cdot 10^7$$

$$N := 9000$$

$$\zeta := 0.707 \qquad \tau_1 := \left(\frac{\omega_n^{2}\cdot N}{K_d\cdot K_v}\right)^{-1} \qquad \tau_2 := \frac{2\cdot\zeta}{\omega_n}$$

$$G_{olz}(z,\omega_n,\zeta) := (\omega_n\cdot T_s)^2\cdot\left[\frac{z\cdot\left(0.5+\frac{1}{T_s}\cdot\frac{2\cdot\zeta}{\omega_n}\right)+\left(0.5-\frac{1}{T_s}\cdot\frac{2\cdot\zeta}{\omega_n}\right)}{(z-1)^2}\right]$$

$$Np := 35$$

$$np := 0..Np-1$$

$$fsw_{np} := 10^{4.9\cdot\left(0.20+0.80\cdot\frac{np}{Np-1}\right)}$$

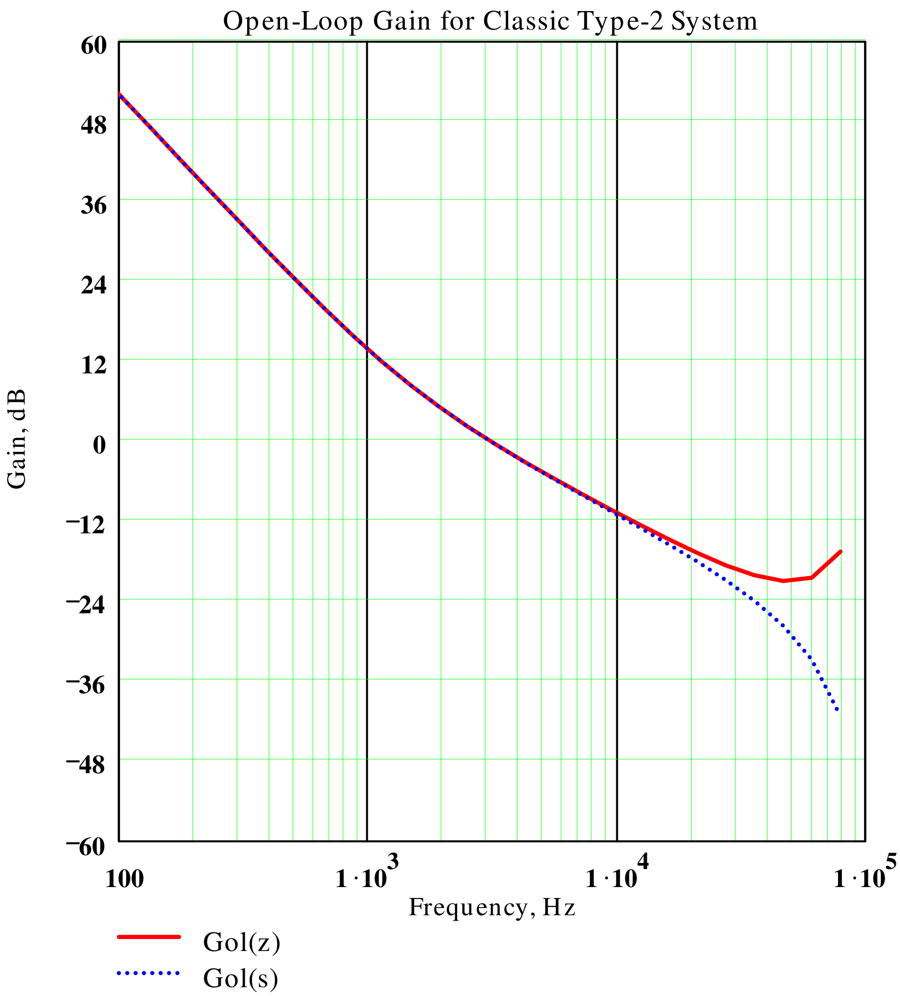
$$Gmz_{np} := 10\cdot\log\left[\left(\left|G_{olz}\left(e^{jx\cdot 2\cdot\pi\cdot fsw_{np}\cdot T_s},\omega_n,\zeta\right)\right|\right)^2\right]$$

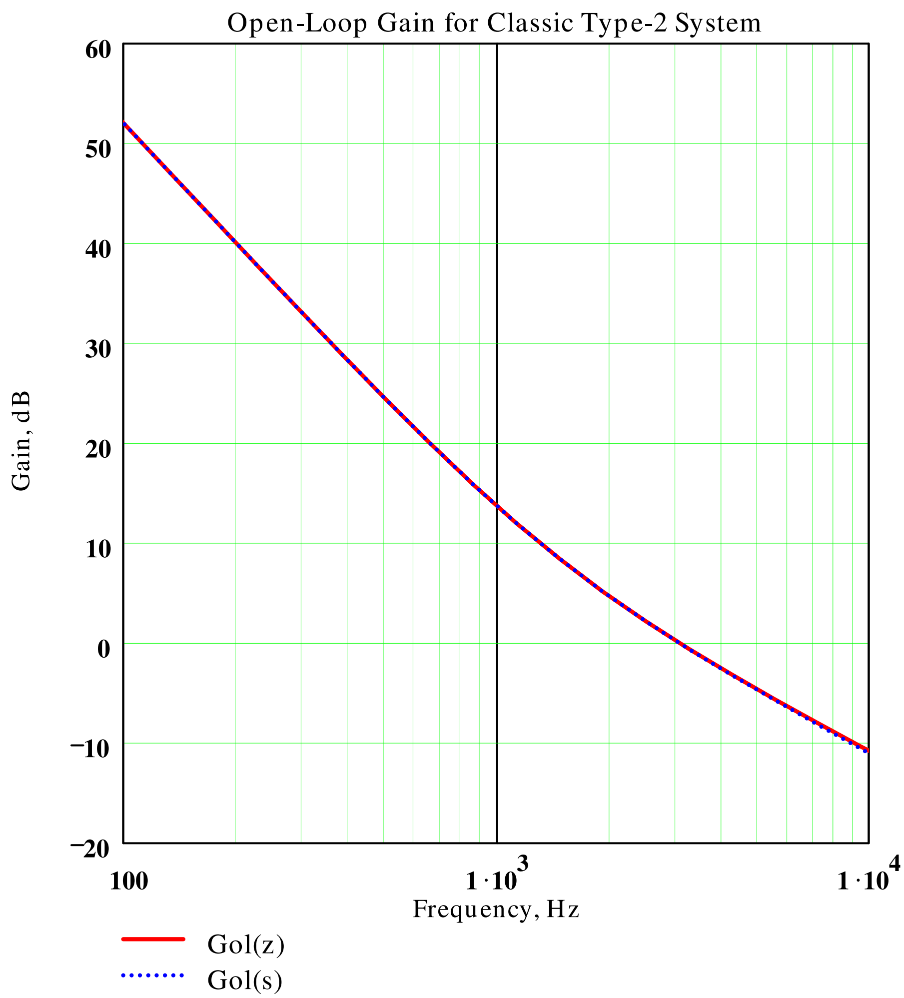
$$G_{ols}(s,\omega_n,\zeta) := \left(\frac{\omega_n}{s}\right)^2\cdot\left(\frac{1-e^{-s\cdot T_s}}{s\cdot T_s}\right)\cdot\left(1+\frac{2\cdot\zeta\cdot s}{\omega_n}\right)$$

$$Gms_{np} := 10\cdot\log\left[\left(\left|G_{ols}(jx\cdot 2\cdot\pi\cdot fsw_{np},\omega_n,\zeta)\right|\right)^2\right]$$

$$Gcl1_{np} := 10\cdot\log\left[\left[\left|\frac{G_{ols}(jx\cdot 2\cdot\pi\cdot fsw_{np},\omega_n,\zeta)}{1+\left[G_{olz}\left[e^{(jx\cdot 2\cdot\pi\cdot fsw_{np}\cdot T_s)},\omega_n,\zeta\right]\right]}\right|\right]^2\right]$$

$$Gcl2_{np} := 10\cdot\log\left[\left[\left|\frac{1}{1+\left[G_{olz}\left[e^{(jx\cdot 2\cdot\pi\cdot fsw_{np}\cdot T_s)},\omega_n,\zeta\right]\right]}\right|\right]^2\right]$$


Open-Loop Gain for Classic Type-2 System
60
48
36
24
12
0
−12
−24
−36
−48
−60
Gain, dB
100
1·10^3
1·10^4
1·10^5
Frequency, Hz
Gol(z)
Gol(s)


Open-Loop Gain for Classic Type-2 System
Gain, dB
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50
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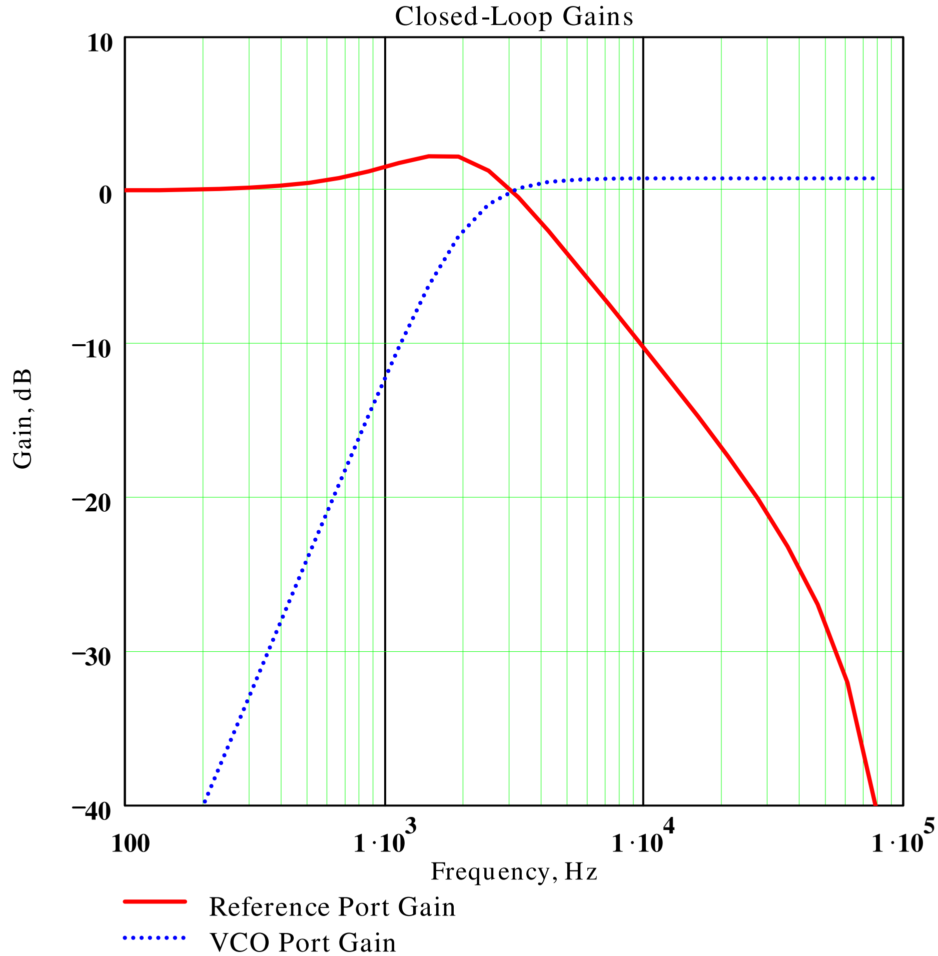
−20
100
1·10^3
1·10^4
Frequency, Hz
Gol(z)
Gol(s)



$$\mathrm{Gmarz}(F_n, \zeta, F_s) := \left| \begin{array}{l} \mathrm{Gm} \leftarrow -20 \cdot \log\left(\zeta \cdot 2 \cdot \pi \cdot F_n \cdot F_s^{-1}\right) \\ \mathrm{Gm} \leftarrow 0 \quad \text{if } \left(2 \cdot \pi \cdot F_n \cdot F_s^{-1}\right) \geq 4 \cdot \zeta \end{array} \right.$$

$$\mathrm{Gmarz}\left(10000, 0.707, 10^5\right) = 7.048$$

$F_{180} := \frac{F_s}{2\cdot\pi}$ $T_s = 1\times 10^{-5}$ $jx = i$

$\omega_{180} := 2\cdot\pi\cdot F_{180}$

Given

$\arg\left(G_{ols}\left(jx\cdot\omega_{180}, \omega_n, \zeta\right)\right) = -\pi$ $\omega_{180} > 2\cdot\pi\cdot\frac{F_s}{6}$

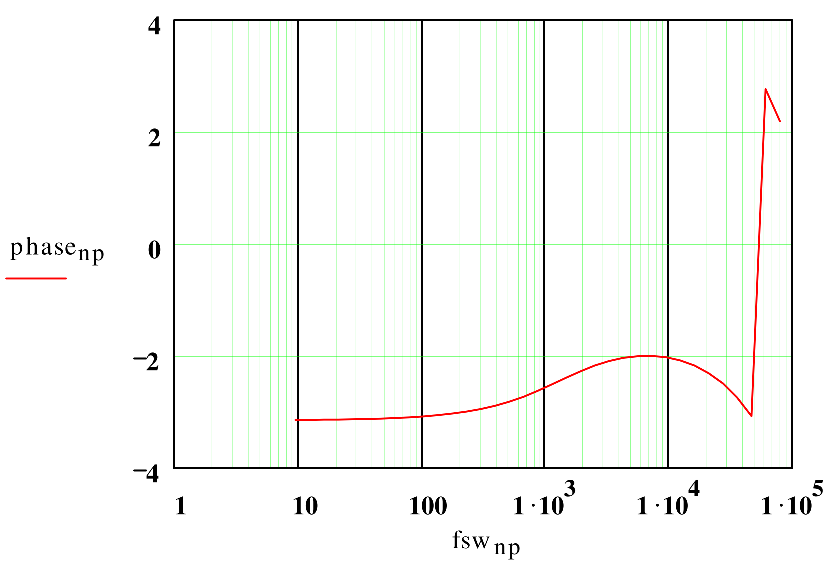
$GmarF(\omega_n, \zeta) := Find(\omega_{180})$

$$\frac{GmarF(2\cdot\pi\cdot 1000, 0.707)}{2\cdot\pi} = 4.955\times 10^{4}$$

$$Gmars(\omega_n, \zeta) := \begin{vmatrix} \omega_{180} \leftarrow GmarF(\omega_n, \zeta) \\ Gmar \leftarrow -10\cdot\log\left[\left[\left(\left|G_{ols}\left(jx\cdot\omega_{180}, \omega_n, \zeta\right)\right|\right)^2\right]\right] \end{vmatrix}$$

$Gmars(2\cdot\pi\cdot 10000, 0.707) = 13.079$

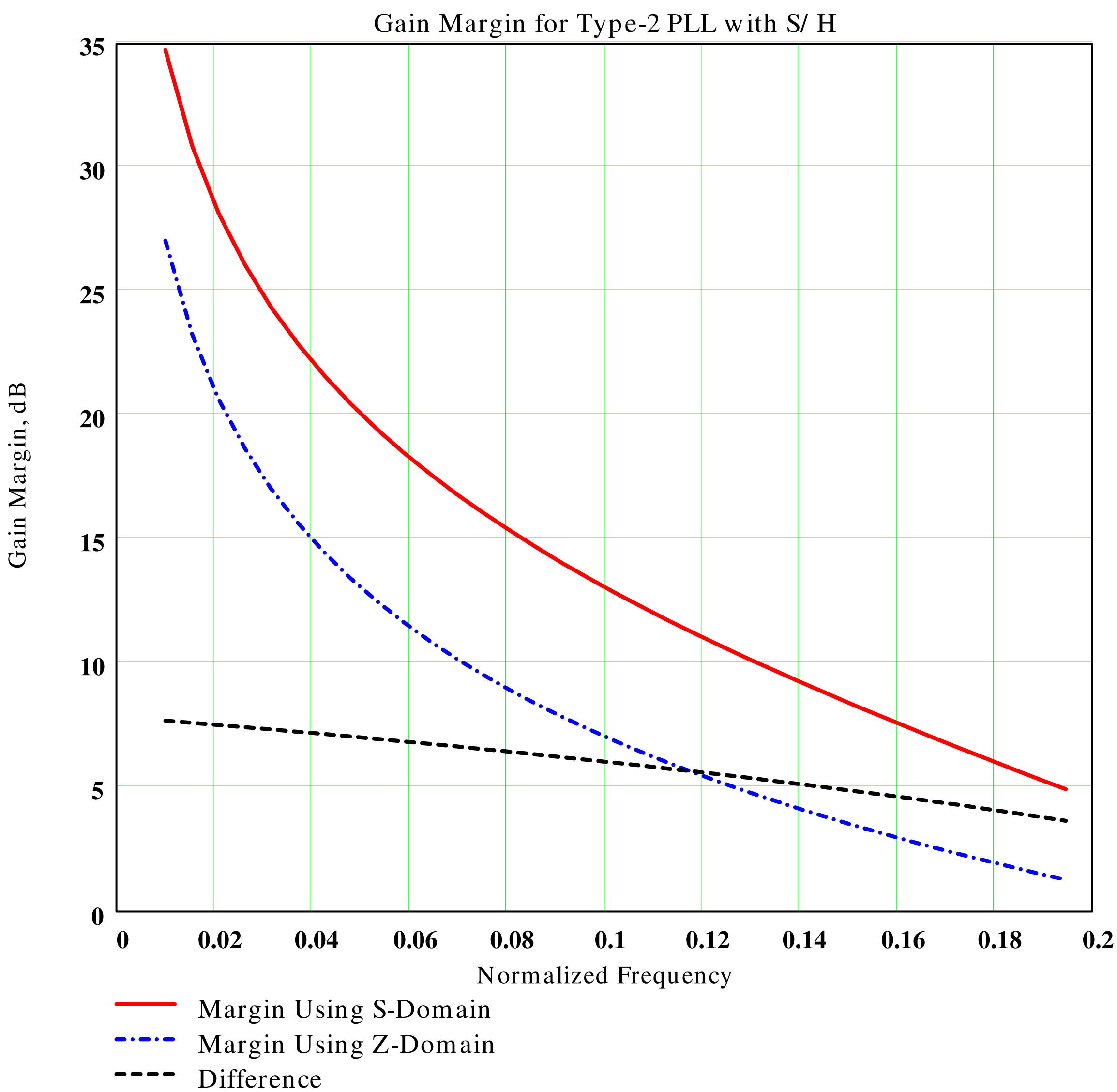
$phase_{np} := \arg\left(G_{ols}\left(jx\cdot 2\cdot\pi\cdot fsw_{np}, \omega_n, \zeta\right)\right)$



$$fnswp_{np} := \frac{np}{Np} \cdot 19000 + 1000$$

$$Gm1_{np} := Gmars\left(2 \cdot \pi \cdot fnswp_{np}, 0.707\right)$$

$$Gm2_{np} := Gmarz\left(fnswp_{np}, 0.707, F_s\right) \qquad \delta_{np} := Gm1_{np} - Gm2_{np}$$



Phase Noise Probability Distribution Function for Sine Wave in AWGN

Ref: "On the Phase Error Distribution of an Open Loop Phase Estimator," 1988 IEEE Intl Conf Comm, W. Hagmann, J. Habermann

By J. Crawford 4 November 1992 Updated 12 March 2004

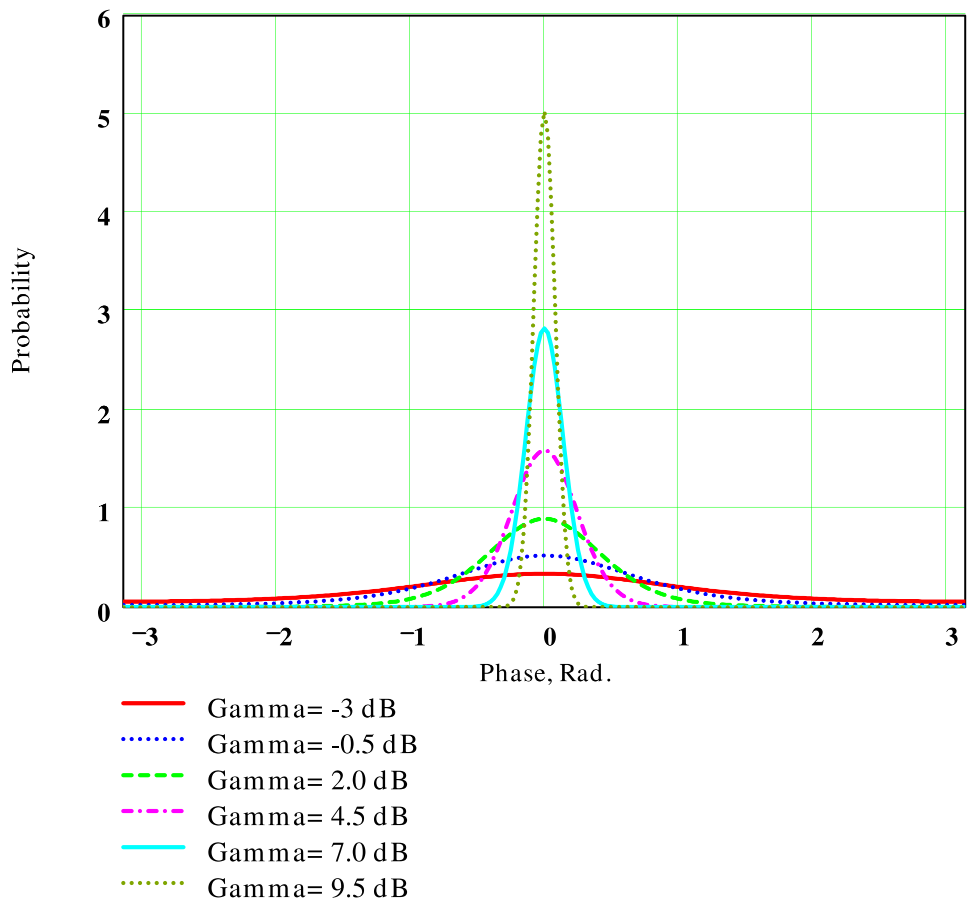
$ii := 0 .. 250$

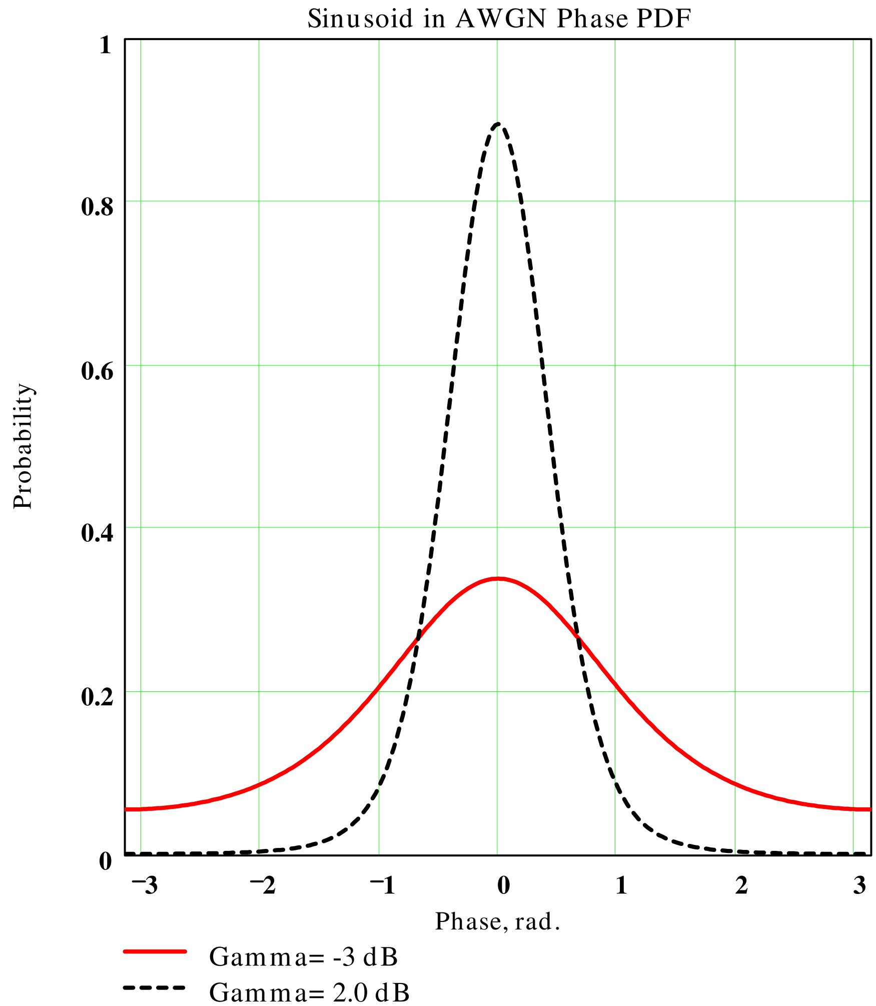
$\gamma := 5.0$ Here, $\gamma := \sqrt{SNR}$

$\theta_{ii} := -\pi + 2 \cdot \frac{\pi}{250} \cdot ii$

$$p(\alpha, \gamma) := \frac{e^{-\gamma^2}}{2 \cdot \pi} \cdot \left[1 + \gamma \cdot \sqrt{\pi} \cdot \cos(\alpha) \cdot e^{\left(\gamma^2 \cdot \cos(\alpha)^2\right)} \cdot (1 + \mathrm{erf}(\gamma \cdot \cos(\alpha))) \right]$$

$jj := 0 .. 5$ $\gamma_{jj} := 10^{0.1 \cdot (-3.0 + jj \cdot 2.5)}$ $y_{jj, ii} := p\left(\theta_{ii}, \gamma_{jj}\right)$



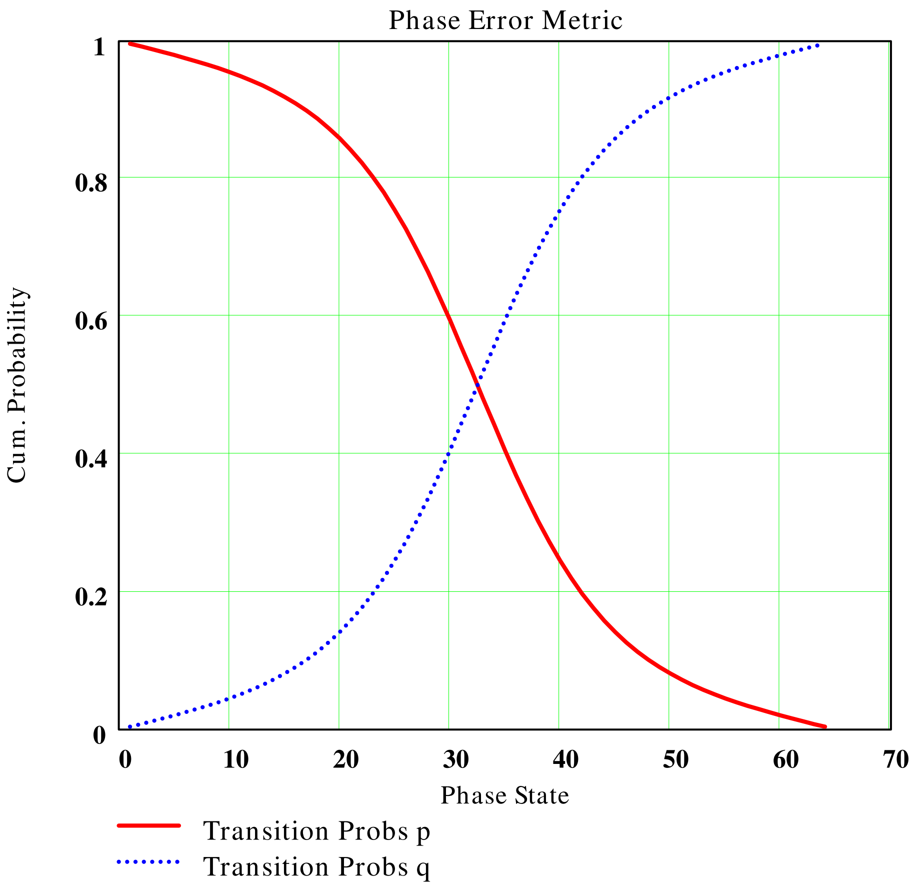

Sinusoid in AWGN Phase PDF
Probability
1
0.8
0.6
0.4
0.2
0
−3
−2
−1
0
1
2
3
Phase, rad.
Gamma= -3 dB
Gamma= 2.0 dB

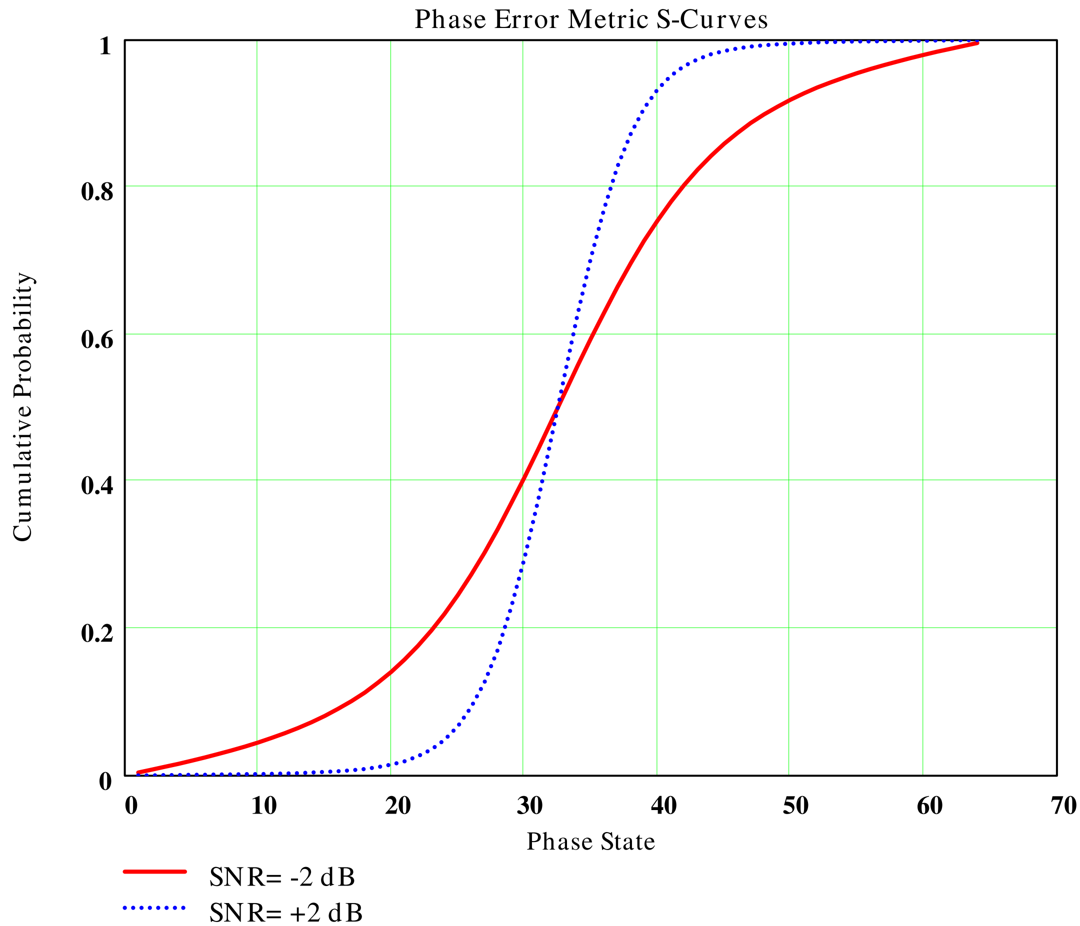
$$\mathrm{pcum}(\theta,\gamma) := \int_{\theta}^{\pi} p(\alpha,\gamma)\,d\alpha \qquad N_{states} := \mathbf{64} \qquad jj := \mathbf{1} .. N_{states} \qquad \gamma_1 := \mathbf{10}^{\mathbf{0.1}\cdot(-\mathbf{2})}$$

$$\gamma_2 := \mathbf{10}^{\mathbf{0.1}\cdot\mathbf{2}}$$

$$\mathrm{phase}_{jj} := \left(\frac{jj}{N_{states}+\mathbf{1}}\cdot\pi\cdot\mathbf{2}\right) - \pi$$

$$p1_{jj} := \mathrm{pcum}\left(\mathrm{phase}_{jj},\gamma_1\right) \qquad q1_{jj} := \mathbf{1} - p1_{jj} \qquad p2_{jj} := \mathrm{pcum}\left(\mathrm{phase}_{jj},\gamma_2\right) \qquad q2_{jj} := \mathbf{1} - p2_{jj}$$

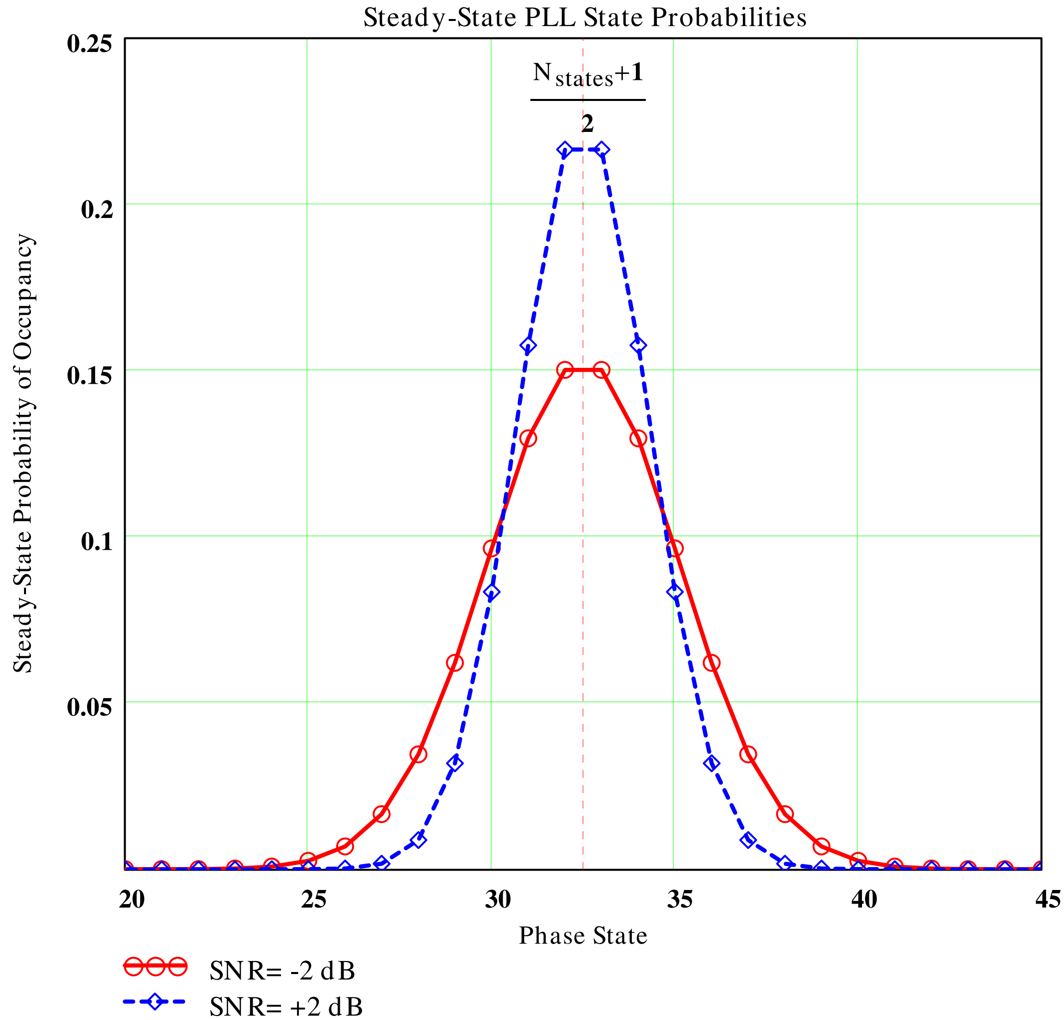




$$S1_1 := \left(1 + \sum_{k=2}^{N_{states}} \prod_{i=1}^{k-1} \frac{p1_i}{q1_{i+1}}\right)^{-1} \qquad kk := 2 .. N_{states}$$

$$S1_{kk} := S1_1 \cdot \prod_{ii=1}^{kk-1} \frac{p1_{ii}}{q1_{ii+1}}$$

$$S2_1 := \left(1 + \sum_{k=2}^{N_{states}} \prod_{i=1}^{k-1} \frac{p2_i}{q2_{i+1}}\right)^{-1} \qquad S2_{kk} := S2_1 \cdot \prod_{ii=1}^{kk-1} \frac{p2_{ii}}{q2_{ii+1}}$$



$$var1 := \sum_{kk=1}^{N_{states}} \left(kk - \frac{N_{states}+1}{2}\right)^2 \cdot S1_{kk}$$

$$\sqrt{var1} \cdot \frac{360}{N_{states}} = 14.694$$

$$var2 := \sum_{kk=1}^{N_{states}} \left(kk - \frac{N_{states}+1}{2}\right)^2 \cdot S2_{kk}$$

$$\sqrt{var2} \cdot \frac{360}{N_{states}} = 9.942$$

Compute Tracking Variance Versus SNR and Number of States

$$\mathrm{TrackVar}(SNR_{dB}, N) := \begin{array}{|l} \text{for } ss \in 1..N \\ \quad \begin{array}{|l} phase_{ss} \leftarrow \left(\frac{ss}{N+1}\cdot 2\cdot\pi\right) - \pi \\ p_{ss} \leftarrow pcum\left(phase_{ss}, 10^{0.1\cdot SNR_{dB}}\right) \\ q_{ss} \leftarrow 1 - p_{ss} \end{array} \\ S_1 \leftarrow \left(1 + \sum_{k=2}^{N} \prod_{i=1}^{k-1} \frac{p_i}{q_{i+1}}\right)^{-1} \\ \text{for } kk \in 2..N \\ \quad S_{kk} \leftarrow S_1 \cdot \prod_{ii=1}^{kk-1} \frac{p_{ii}}{q_{ii+1}} \\ \mu \leftarrow \sum_{kk=1}^{N} kk\cdot S_{kk} \\ var \leftarrow \sum_{kk=1}^{N} (kk-\mu)^2\cdot S_{kk} \\ stddevdeg \leftarrow \sqrt{var}\cdot\frac{360}{N} \end{array}$$

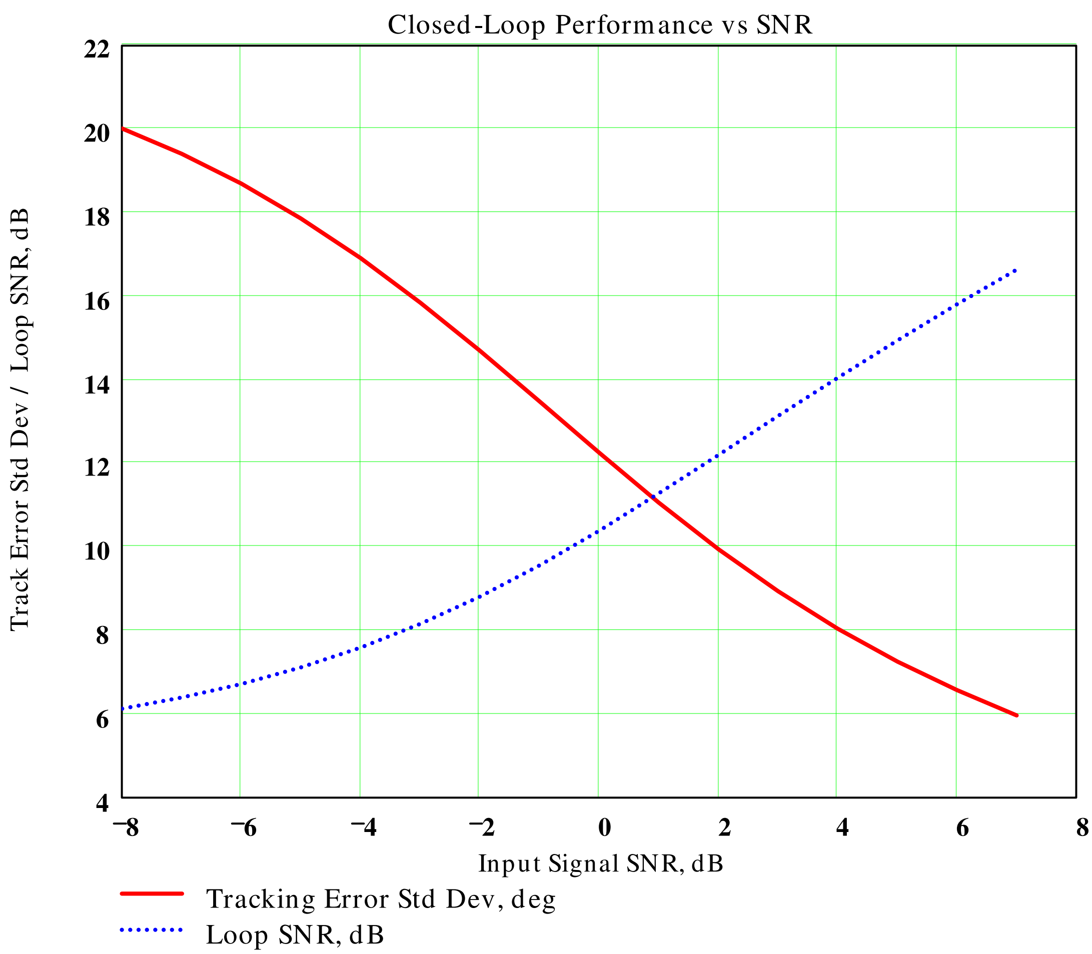
$$\mathrm{TrackVar}(2, 64) = 9.942$$

$rr := 0 .. 15$

$\rho_{rr} := -8 + rr$

$trk_{rr} := TrackVar(\rho_{rr}, 64)$

$$SNRL_{rr} := 10 \cdot \log\left[\frac{1}{2} \cdot \left(trk_{rr} \cdot \frac{\pi}{180}\right)^{-2}\right]$$

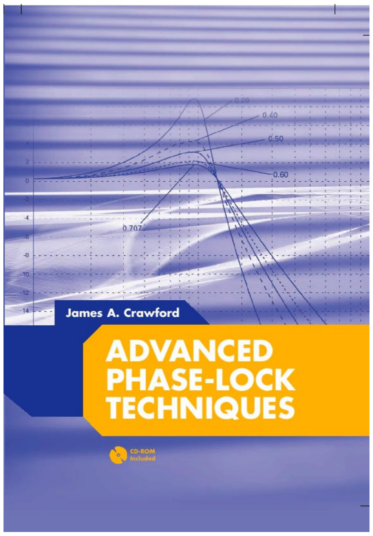


EXTRA: Mean-Time-to-Slip

$$\text{SlipTime}(p,q) := \sum_{k=1}^{N_{states}} \sum_{i=1}^{k} \frac{1}{q_i} \cdot \prod_{j=i}^{k} \left(\frac{q_j}{p_j}\right)$$

$$\text{SlipTime}(p1,q1) = 1.668 \times 10^{33}$$

$$\text{SlipTime}(p2,q2) = 1.015 \times 10^{65}$$



Advanced Phase-Lock Techniques

James A. Crawford

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2	*Design Notes* A compilation of design notes and formulas that are developed in details separately in the text. Includes an exhaustive list of closed-form results for the classic type-2 PLL, many of which have not been published before.	44
3	*Fundamental Limits* A detailed discussion of the many fundamental limits that PLL designers may have to be attentive to or else never achieve their lofty performance objectives, e.g., Paley-Wiener Criterion, Poisson Sum, Time-Bandwidth Product.	38
4	*Noise in PLL-Based Systems* An extensive look at noise, its sources, and its modeling in PLL systems. Includes special attention to $1/f$ noise, and the creation of custom noise sources that exhibit specific power spectral densities.	66
5	*System Performance* A detailed look at phase noise and clock-jitter, and their effects on system performance. Attention given to transmitters, receivers, and specific signaling waveforms like OFDM, M-QAM, M-PSK. Relationships between EVM and image suppression are presented for the first time. The effect of phase noise on channel capacity and channel cutoff rate are also developed.	48
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